2000-09

Optimization of procurement scheduling for Major Defense Acquisition programs

Humpert, Donald E.

Monterey, California. Naval Postgraduate School

http://hdl.handle.net/10945/7755

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

OPTIMIZATION OF PROCUREMENT SCHEDULING FOR MAJOR DEFENSE ACQUISITION PROGRAMS

by

Donald E. Humpert

September 2000

Thesis Advisor:	Alan R. Washburn
Second Reader:	Timothy P. Anderson

Approved for public release; distribution is unlimited.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	September 2000	Master's Thesis

4. TITLE AND SUBTITLE:
Optimization of Procurement Scheduling for Major Defense Acquisition Programs

5. FUNDING NUMBERS

6. AUTHOR(S)
Humpert, Donald E.

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
OUSD(AT&L) / ARA
Pentagon Room 3D161
Washington DC 20301-3000

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(maximum 200 words)*
As the Defense Acquisition Executive (DAE), the Under Secretary of Defense for Acquisitions, Technology and Logistics has full responsibility for supervising the performance of the DoD Acquisition System. A challenge to the DAE is in determining the most efficient allocation of funding in procuring of over eighty Major Defense Acquisition Programs. This thesis develops six different cost functions based on the Unit Theory learning curve model for estimating the cost of each of these MDAP systems. The most suitable of these adds an annual overhead component to the cost modeled by the learning effect. This function is implemented in an integer-linear optimization model, the Procurement Scheduling Optimization Model (PSOM). PSOM allows the planner to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest full rate production (FRP) start periods; and low rate initial production (LRIP) costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints, finding the optimal quantity of each system to be procured each year of the planning horizon. This thesis models the cost of seventeen of the MDAP systems and optimally schedules them over an eighteen year planning horizon. PSOM can easily be expanded to include all eighty-plus MDAP systems. PSOM is a tool available to acquisition planners and decision makers to assist in optimally allocating procurement funding.

14. SUBJECT TERMS
Acquisitions, Procurement, Cost Estimation, Learning Curve, MDAP, Optimization, Linear Program, Integer-Linear Program

15. NUMBER OF PAGES
98

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. 239-18

THIS PAGE INTENTIONALLY LEFT BLANK

Approved for public release; distribution is unlimited.

OPTIMIZATION OF PROCUREMENT SCHEDULING FOR MAJOR DEFENSE ACQUISITION PROGRAMS

Donald E. Humpert
Major, United States Marine Corps
B.A., University of Colorado, 1987
M.A., University of Redlands, 1994

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 2000

THIS PAGE INTENTIONALLY LEFT BLANK

ABSTRACT

As the Defense Acquisition Executive (DAE), the Under Secretary of Defense for Acquisitions, Technology and Logistics has full responsibility for supervising the performance of the DoD Acquisition System. A challenge to the DAE is in determining the most efficient allocation of funding in procuring of over eighty Major Defense Acquisition Programs. This thesis develops six different cost functions based on the Unit Theory learning curve model for estimating the cost of each of these MDAP systems. The most suitable of these adds an annual overhead component to the cost modeled by the learning effect. This function is implemented in an integer-linear optimization model, the Procurement Scheduling Optimization Model (PSOM). PSOM allows the planner to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest full rate production (FRP) start periods; and low rate initial production (LRIP) costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints, finding the optimal quantity of each system to be procured each year of the planning horizon. This thesis models the cost of seventeen of the MDAP systems and optimally schedules them over an eighteen year planning horizon. PSOM can easily be expanded to include all eighty-plus MDAP systems. PSOM is a tool available to acquisition planners and decision makers to assist in optimally allocating procurement funding.

THIS PAGE INTENTIONALLY LEFT BLANK

DISCLAIMER

The reader is cautioned that the computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the planner.

THIS PAGE INTENTIONALLY LEFT BLANK

TABLE OF CONTENTS

LIST OF FIGURES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF TABLES

THIS PAGE INTENTIONALLY LEFT BLANK

LIST OF ACRONYMS

AAAV	Advanced Amphibious Assault Vehicle
AFIT	Air Force Institute of Technology
ACAT 1	Acquisition Category 1
AUC	Average Unit Cost
BIFV	Bradley Infantry Fighting Vehicle
DAE	Defense Acquisition Executive
DPP	Defense Program Projection
EELV	Evolved Expendable Launch Vehicle
FRP	Full Rate Production
FYDP	Future Years Defense Program
GAMS	General Algebraic Modeling System
IDA	Institute for Defense Analysis
JSOW	Joint Stand-Off Weapon
LRIP	Low Rate Initial Production
LMP	Algebraic Lot Midpoint
LURP	Linearized-Unit-Rate-Penalty
OUSD(AT&L)	Office of the Under Secretary of Defense (Acquisition, Technology and Logistics)
PSOM	Procurement Scheduling Optimization Model
QDR	Quadrennial Defense Review
SAR	Selected Acquisition Reports
USD(AT&L)	Under Secretary of Defense (Acquisition, Technology and Logistics)

THIS PAGE INTENTIONALLY LEFT BLANK

EXECUTIVE SUMMARY

As the Defense Acquisition Executive, the Under Secretary of Defense for Acquisitions, Technology and Logistics (AT&L) has full responsibility for supervising the performance of the DoD Acquisition System. A phenomenon that the Office of the Under Secretary of Defense (AT&L) has recognized is that the total estimated cost of any given defense system increases as the rate of procurement for that system decreases. In a world of unlimited resources, all systems could be acquired at the maximum rate possible at the lowest possible cost; unfortunately, we live with the reality of a limited annual budget. The decision maker's dilemma is in choosing how to "best" schedule the acquisition of these systems, subject to budget limitations and other requirements. This thesis provides the decision maker with a tool to address this problem.

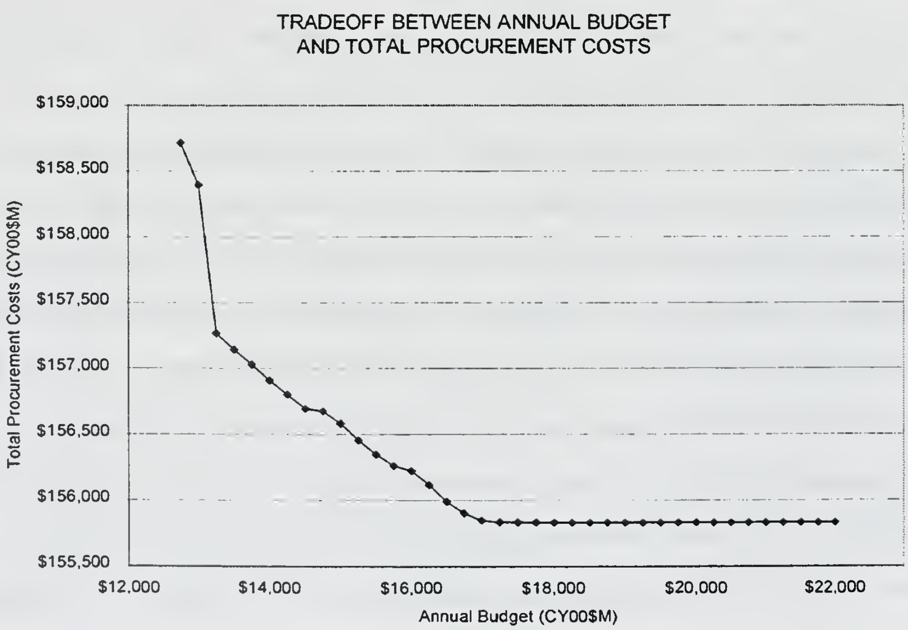
The most critical requirement of such a tool is that it accurately reflect the estimated cost of the systems that are scheduled with it. This thesis uses Selected Acquisition Report (SAR) data for seventeen Major Defense Acquisition Programs to develop six different cost estimating relationships and evaluate their suitability for this purpose. The "Base Model" is simply an expression of basic learning curve theory: each system has a "beginning" price per unit which decreases as more units of the system are produced. The five other cost estimating relationships are excursions from this model. The "Multiplicative-Rate Model," "IDA Rate-Penalty Model," and "Linearized-Unit-Rate-Penalty Model" all include the rate of production as a predictor of unit cost. The "Rate-Change Model" assumes that changes in rate of production from one period to the next contribute to the cost of each system. The "Base + Overhead Model" adds a fixed cost component to the basic learning curve cost for each lot of the system procured. The primary measure of effectiveness (MOE) for comparing each cost estimating relationship is how well their predicted lot costs agree with the SAR data lot costs. By this MOE, the Base + Overhead Model is the best cost estimating relationship of the six.

Implementing each cost function in a spreadsheet tool provides additional insight into its suitability. The cost functions are used to estimate the annual and total procurement costs for the seventeen systems for which we have data. The difference between the modeled total cost and the "true" total cost (calculated from SAR data)

serves as a second MOE. By this MOE, the Base + Overhead Model is again superior. Although the spreadsheet tool can be used to "manually" adjust the procurement plan for each system in search of the least expensive schedule, this is prohibitively tedious. This implies the requirement for an optimizing planning tool.

The Base + Overhead cost function is implemented in an integer-linear program, the Procurement Scheduling Optimization Model (PSOM). PSOM is implemented in the General Algebraic Modeling System (GAMS), and schedules the quantity of each MDAP system to be procured per year over an 18-year planning horizon. PSOM allows the planner to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest full rate production (FRP) start periods; and low rate initial production (LRIP) costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints. Data input to the model requires a working knowledge of GAMS.

PSOM can be used to construct a chart of the efficiency frontier, a plot of the minimum total cost of all systems at varying budget limits. This is built by repeatedly solving PSOM in a loop, with the budget decreasing after each iteration, from an amount in which the constraint is slack to the point at which the model becomes infeasible. The efficiency frontier for the seventeen systems modeled in this thesis is presented below.



Efficiency Frontier for Subset of MDAP Systems

Procurement schedules corresponding to all points above the line are sub-optimal. Schedules corresponding to points below the line are infeasible. The decision maker can use the information in many ways. Assuming that the current schedule is sub-optimal, the decision maker may choose to optimize the schedule for the current budget and thus reduce overall cost; or, given an allowable overall cost, the decision maker may choose to reduce the annual budget available. If the schedule is already optimal, the decision maker will readily appreciate the effect of changing the annual budget limit; the additional cost of a reduction in budget, or the potential long-term savings from an increased budget, are equally apparent.

PSOM can be easily expanded to include all 80+ MDAP systems. Expansion and use of PSOM or a similar optimization model is recommended for use by acquisition planners.

THIS PAGE INTENTIONALLY LEFT BLANK

ACKNOWLEDGMENT

I would like to thank Professor Alan Washburn for his patient guidance through this effort, and for his mentoring throughout my two years at the Naval Postgraduate School. He never hesitated in allowing me his time, always accompanied by valuable advice. Lieutenant Commander Tim Anderson also deserves special thanks. His willingness in sharing his knowledge and enthusiasm towards this topic were truly inspiring.

I would also like to thank: my sponsor, Dr. Nancy Spruill, Principal Deputy Under Secretary of Defense (Acquisition, Technology and Logistics), for her generosity in funding this effort; Philip Rodgers, Deputy Director, Resource Analysis at OUSD(AT&L) for remembering his alma matter in making this project available; Dr. Stephen Balut, Director of the Cost Analysis and Research Division at IDA for his support in providing me with six weeks of priceless "real-world" experience; and Dr. Chuck Weber, the "Master Production Schedule" project leader at IDA for integrating me into his team and providing me with the data I needed for this endeavor.

Finally, I thank my wife Judi, and our three amazing children Brad, Kim and Angela. Their patience throughout this entire postgraduate experience has been truly remarkable. Of all the insights I've gained from my time in Monterey, the most valuable is in realizing once again the importance of family.

THIS PAGE INTENTIONALLY LEFT BLANK

I. INTRODUCTION

In his keynote address to Advanced Program Managers Course 99-02, Dr. Jacques Gansler, Under Secretary of Defense (Acquisition, Technology and Logistics) listed three "vital priorities" for the DoD acquisition community over the next few years (Gansler, 1999):

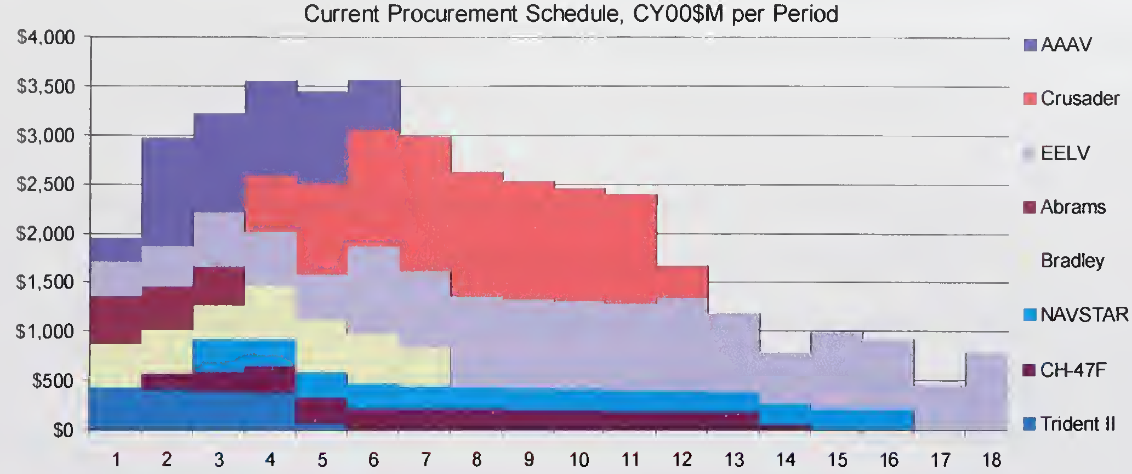
- To equip the early 21st century warfighter with the right equipment to assure our security and withstand any potential threat.
- To accelerate, broaden, and institutionalize our acquisition reform efforts *in order to optimize our limited resources* in providing those weapons. (emphasis added)
- To modernize our logistics systems -- so as to cut costs, infrastructure, and cycle time in support of our 21st century forces.

The intent of the second priority—"*in order to optimize our limited resources*"—provides the basis for this thesis.

A. THE CHALLENGE

A phenomenon that the Office of the Under Secretary of Defense (Acquisition, Technology and Logistics) (OUSD(AT&L)) has recognized is that the total estimated cost of any given system being procured increases as the rate of procurement for that system decreases. As a limited budget forces fewer of each system to be procured per year, the systems must be produced at a lesser rate, over a longer period. Either the rate or length of production, then, can be considered a contributor to the total procurement cost of a system. If not for the limitation of an annual budget and manufacturing constraints, the least costly strategy would be to procure all units of a system in one year—clearly an impractical real-world solution. Nonetheless, even considering annual budget limits and plant capacities, it is a relatively simple task to determine the optimal procurement schedule for a single system. Conceptually, it should also be possible to determine the optimal procurement schedule for all systems in combination. This "master" schedule is much more difficult to find, requiring suitable cost functions to be developed for each system and modeling their interactions. The 2001 Quadrennial Defense Review (QDR) offers an opportunity to take advantage of scheduling efficiencies if they can be

discovered. Figure (1) illustrates hypothetically how rearranging a procurement schedule could reduce overall cost.



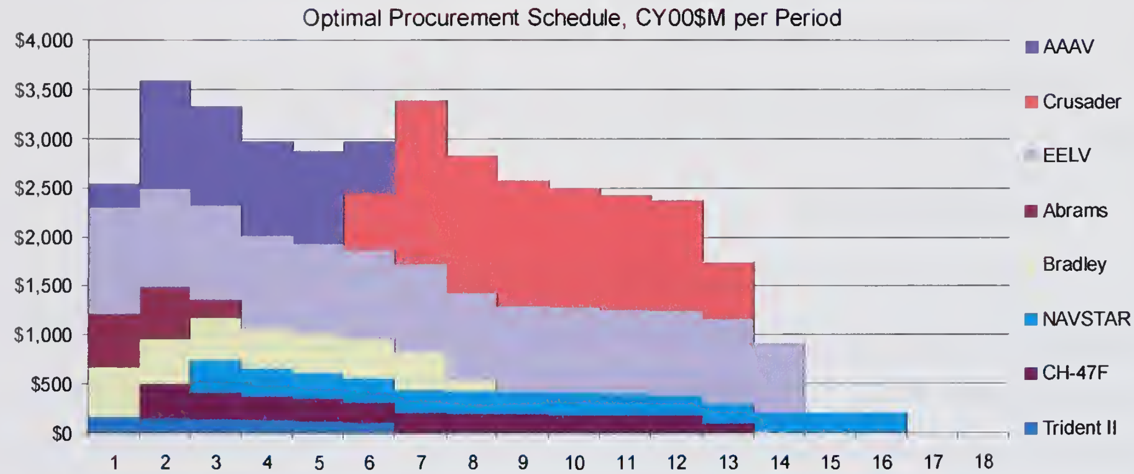


Figure 1. Example of Optimized Schedule

The top chart shows the current procurement schedule for eight MDAP systems. A cost function that incorporates the effects of learning and production rates was used to model the cost of each system. The bottom chart shows the same total quantity of each system, procured over a more efficient schedule. The total cost of all systems in the current schedule is 38,541 CY00$M; the total cost of all systems in the optimal schedule is 37,464 CY00$M. Over a billion dollars is saved.

During each Quadrennial Defense Review (QDR), the DoD reviews its strategy for acquisition of new weapons systems and equipment. Each service and DoD entity argues in support of its priority interests. As the Defense Acquisition Executive, USD(AT&L) has full responsibility for supervising the performance of the DoD Acquisition System. A task of OUSD(AT&L) during the QDR will be to provide an economic framework for these decisions. A minimum cost, optimized procurement

schedule would be an ideal baseline from which the cost of deviations could be readily assessed. Therefore, an OUSD(AT&L) goal is to have developed an optimal Master Production Schedule for the DoD Major Defense Acquisition Programs (MDAP) for use in the 2001 QDR. This schedule will provide the least costly strategy to purchase MDAP systems for the next eighteen years.

In April 1998, OUSD(AT&L) tasked the Institute for Defense Analysis (IDA) to explore the use of optimization technology for long range defense acquisition planning (Weber, 1999). IDA was asked to focus on the development of such a Master Production Schedule for Acquisition Category (ACAT 1) systems that would meet the following requirements:

- Incorporate the approximately 80 Acquisition Category 1 (ACAT 1) systems in development or planned for the future.
- Cover the 18-year Defense Program Projection (DPP) planning horizon.
- Serve as an aggregate-level planning tool. The model would be developed at the system level, not being concerned with the detailed modeling of systems' components and sub-components.
- Reflect long-term planning issues, avoiding short-term scheduling issues such as assembly line balancing, potential strikes, etc.
- Reflect existing peacetime conditions to the extent reflected by the DPP (not concerned with wartime attrition).
- Use procurement cost functions for systems that reflect as *realistically as possible* the various factors affecting their values.

Fulfilling the last of these requirements represents one of the most challenging aspects of this effort. Since the cost function for each system is required in both the objective function and budgetary constraints of an optimization model, the model will be sensitive to the correctness of these cost estimating relationships.

The optimization model will seek to minimize the total cost of all systems over all years, subject to constraints. The most conspicuous of these constraints is the annual budget. Other constraints include: the requirement to meet the quantity demanded for each system by certain years; limitations on minimum and maximum production rates; and the desire to maintain production stability by forbidding breaks in production. The

model will also allow the user to specify startup and shutdown years, and allow the input of Low Rate Initial Production (LRIP) allowances. Figure (2) illustrates a typical acquisition schedule for a single hypothetical system.

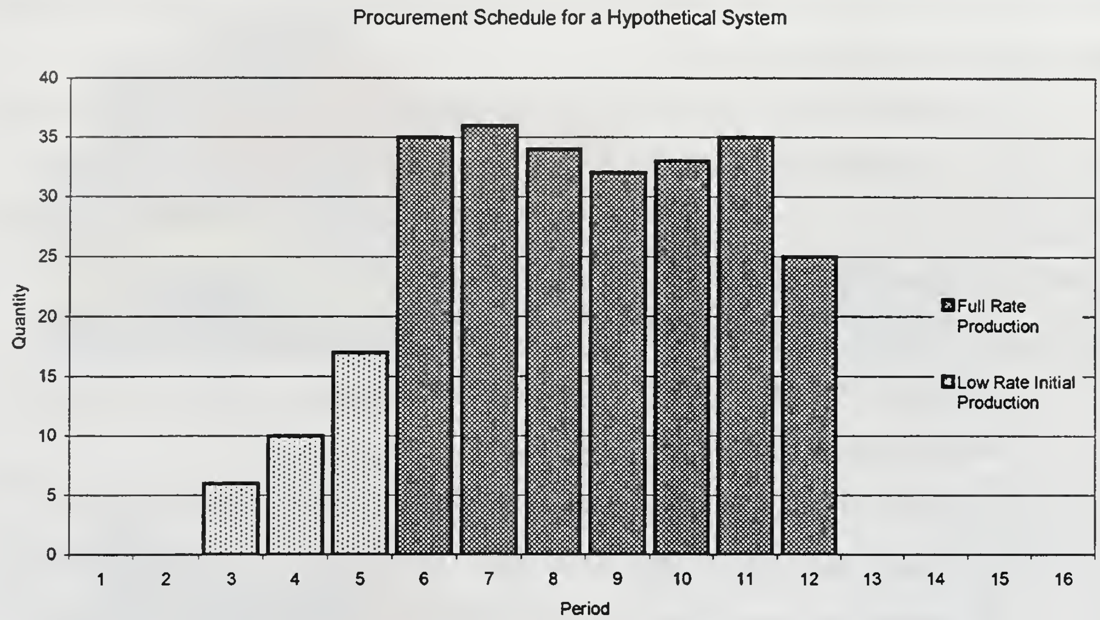


Figure 2. Hypothetical Procurement Schedule for a System
A hypothetical acquisition schedule for a system starting full rate production (FRP) during period six. Quantities produced during low rate initial production (LRIP) are given as data. There are no production breaks once started, and LRIP immediately precedes FRP.

B. PURPOSE

This thesis develops the cost functions, spreadsheet scheduling tools, and a scalable optimization model, for a subset of the MDAP systems that IDA has been tasked with including in their model. The systems in the subset, although not selected at random, were selected to represent the whole population of MDAP systems. The following seventeen systems were selected:

1. Advanced Amphibious Assault Vehicle (AAAV)
2. Abrams Tank upgrade
3. Bradley Infantry Fighting Vehicle (BIFV) upgrade
4. C-17 Aircraft
5. CH-47 F Helicopter Upgrade
6. Crusader Self Propelled Howitzer with Resupply Vehicle

7. DDG 51 Guided Missile Destroyer
8. Evolved Expendable Launch Vehicle (EELV) Space Launch Vehicle
9. F/A-18 E/F Aircraft
10. Joint Standoff Weapon (JSOW)
11. Minuteman III Intercontinental Ballistic Missile
12. NAVSTAR Global Positioning Satellite
13. MV-22 Osprey Tilt-rotor Aircraft
14. SSN 774 Virginia Class Submarine
15. Standard Missile 2
16. T-45 TS Aircraft
17. Trident II Ballistic Missile

The following additional objectives are accomplished:

- Modeling six cost functions, to include comparisons between the models regarding their respective strengths and weaknesses.
- Formulation of a mixed integer program in the GAMS algebraic modeling language using the most appropriate cost function, with the capability to be easily expanded by IDA or OUSD(AT&L) to optimize all eighty ACAT 1 systems.

C. THESIS ORGANIZATION

Chapter II presents the six cost functions. The characteristics of each, the assumptions underlying them, and the data analysis which determines the respective system parameters for each function is given. The use of spreadsheet scheduling tools that use the cost functions developed is explored in Chapter III. The optimization model assumptions are listed, the formulation is offered, its implementation is described and its output is presented in Chapter IV. Chapter V details analysis of the optimization model output and conclusions.

THIS PAGE INTENTIONALLY LEFT BLANK

II. COST FUNCTIONS

Arguably the most crucial step in creating a procurement scheduling tool is to derive a sound cost estimating relationship for each system being modeled. The cost function must as accurately as possible fit available data, while its behavior must agree with our intuition as to what is "correct."

A. DATA

The data required to estimate the cost functions of each system consist of the lot quantities and lot costs of each system, from beginning to end of procurement. The data for all systems but the Crusader and AAAV was obtained from their respective Selected Acquisition Reports (SAR). The SAR data was the most readily available, since IDA maintains copies of them.

A second source of this data is the Procurement Annex of the Future Years Defense Program (FYDP), which is a detailed five-year projection of procurements. This data mirrors that which is reported in the SARs. Since both the Crusader and AAAV are currently still in the RDT&E phase, however, neither their SARs nor the FYDP contained their appropriation data.

A third source of the required data is the DPP, which is less detailed but spans eighteen years. The DPP contains the most accurate data available for systems with incomplete SAR or FYDP data. Both the Crusader and AAAV data was extracted from this source. For each system, the unit cost was recalculated in terms of year 2000 dollars. Appendix A shows the data used for each system.

B. METHODOLOGY

1. Cost Function Development

The principal characteristic of a cost function is that as more units of an item are produced, the average unit cost of that item decreases. In his book *The Cost Analyst's Companion*, David Lee relates that this phenomenon is called "cost progress" and that the cost-quantity relation for a given system in production is known as the "cost-progress curve," or "learning curve."

One of the most commonly used models of cost progress is the unit theory model. The form of this model is

$$C(Q) = T_1 Q^{\beta} \tag{1}$$

where $C(Q)$ is the cost of the Q^{th} unit; T_1 and β are constants. The constant T_1 is the theoretical cost of the first unit produced. The constant β represents the "cost analyst's slope" of the curve, or "slope of the learning curve," defined by

$$S \equiv \frac{Cost\ at\ quantity\ 2Q}{Cost\ at\ quantity\ Q} \tag{2}$$

or

$$S = 2^{\beta} \tag{3}$$

The constant β is assumed to be negative, or the cost or each successive unit would increase, rather that decrease. All of the models developed for use in the scheduling tools are derived from the Unit Theory Model. Figure (3) shows a typical learning curve for a system with theoretical first unit cost T_1 of \$10 and a learning curve slope of 80%.

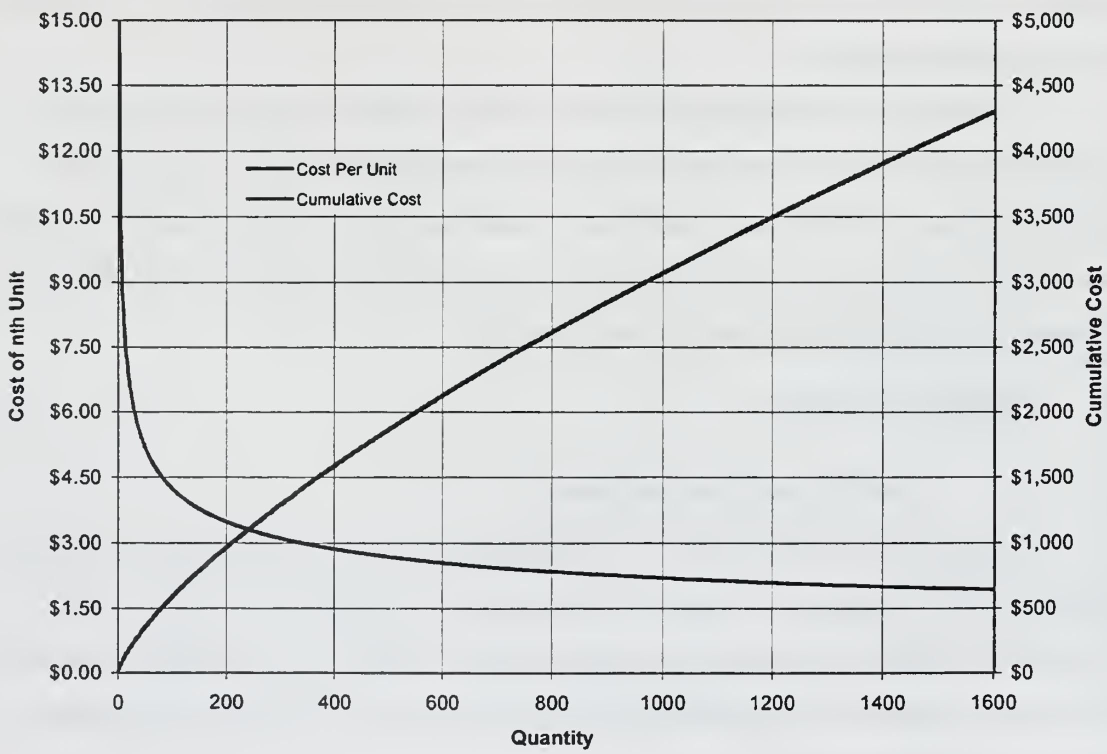


Figure 3. Typical Learning Curve and Cumulative Cost Curve for a Hypothetical System. First unit cost, T_1, is \$10 and learning curve slope of 80% (β = -0.322). The unit cost of the Q^{th} unit is measured on the left axis, the cumulative cost to the Q^{th} unit on the right axis.

2. Fitting the Data

The parameters of each cost function are estimated by fitting them to the data using least-squares techniques. When possible, data are transformed to log-space, and parameters are determined by linear regression. Otherwise, the nonlinear solver resident in Excel is used to find the best fit.

For five of the six cost functions developed herein, the dependent variable of the function is the cost of the Q^{th} unit, $C(Q)$. Since the data is given for annual lots, however, the dependent variable to be used in place of $C(Q)$ is the *average unit cost (AUC),* which is simply defined as

$$AUC = \frac{TOTAL\ LOT\ COST}{LOT\ SIZE} \tag{4}$$

The independent variable to be used in place of Q is the *algebraic lot midpoint (LMP),* the theoretical unit whose cost is equal to the AUC for that lot on the learning curve. The LMP can be approximated using the following rules (AFIT, 1997):

For the first lot (the lot starting at unit 1):

If Lot Size < 10, then LMP = Lot Size/2

If Lot Size $\geq$ 10, then LMP = Lot Size/3

For all other lots:

$$LMP = \frac{F + L + 2\sqrt{F * L}}{4} \tag{5}$$

where F is the first unit number in a lot, and L is the last unit number in a lot.

The sixth cost function is fitted directly to the lot cost and lot quantity, so that expressing the data in terms of unit cost is unnecessary.

3. Filtering the Data

An important issue that must be addressed when considering cost functions that include *rate of production* is the treatment of the first and last production periods. Data typically shows a relatively smaller quantity of a system that is produced in the first and last periods. One of two interpretations must be assumed: either 1) the rate of production is lower in these years; or 2) the first and last lots are not produced over a full year. It is more realistic to expect that the second of these assumptions is correct. Therefore, since the fractional year is not given in the database, including the first and last production

periods in the data used for analysis is not usually appropriate for rate-based cost functions. For models of this type, we drop the last production period from our data. Unless the first production period is obviously a full lot, we drop it from the data also and model it as LRIP.

4. Measures of Effectiveness

A traditional measure of effectiveness for a linear or nonlinear model is the Coefficient of Determination, R^2, defined as

$$R^2 = 1 - \frac{(RESIDUAL\ SS)}{(CORRECTED\ TOTAL\ SS)} \tag{6}$$

where

$$RESIDUAL\ SS = \sum \left(Y_i - \hat{Y}_i\right)^2 \tag{7}$$

$$CORRECTED\ TOTAL\ SS = \sum \left(Y_i - \overline{Y}_i\right)^2 \tag{8}$$

In principle, the R^2 could be negative if the model fits worse than the mean does. We calculate all R^2 in unit-space, and adjust them to take into account the complexity of the model relative to the complexity of the data as follows

$$R^2_{adj} = R^2 - \frac{K-1}{n-K}\left(1 - R^2\right) \tag{9}$$

where n is sample size and K is the number of parameters in the model (Hamilton, 1992).

Since our interest is in how well the cost functions estimate the systems' lot costs, it is the R^2_{adj} of the fit of the *lot costs* that is our principal measure of effectiveness for a function.

C. BASE MODEL

1. Characteristics and Assumptions

The "Base Model" is simply the Unit Theory model presented in equation (1). As previously discussed, this model captures only the effect of learning on system cost. This model ignores the production rate. Therefore, whether all units are produced in the first period, or production is spread over several periods, the total system cost is the same.

2. Model Fit

The Base Model is the simplest function to fit. Since equation (1) is easily transformed to

$$\ln C(Q) = \ln T_1 + \beta \ln Q \tag{10}$$

it is a simple matter to perform a linear regression to obtain the coefficients T_1 and β. With these coefficients, the annual costs of each system are readily determined. The median R^2_{adj} of the fit of the annual costs for this model over all systems is 0.75; the mean is 0.65. Parameters for each system and respective R^2_{adj} values are displayed in Appendix B. Figure (4) shows the data and fitted values for the BIFV unit and lot costs.

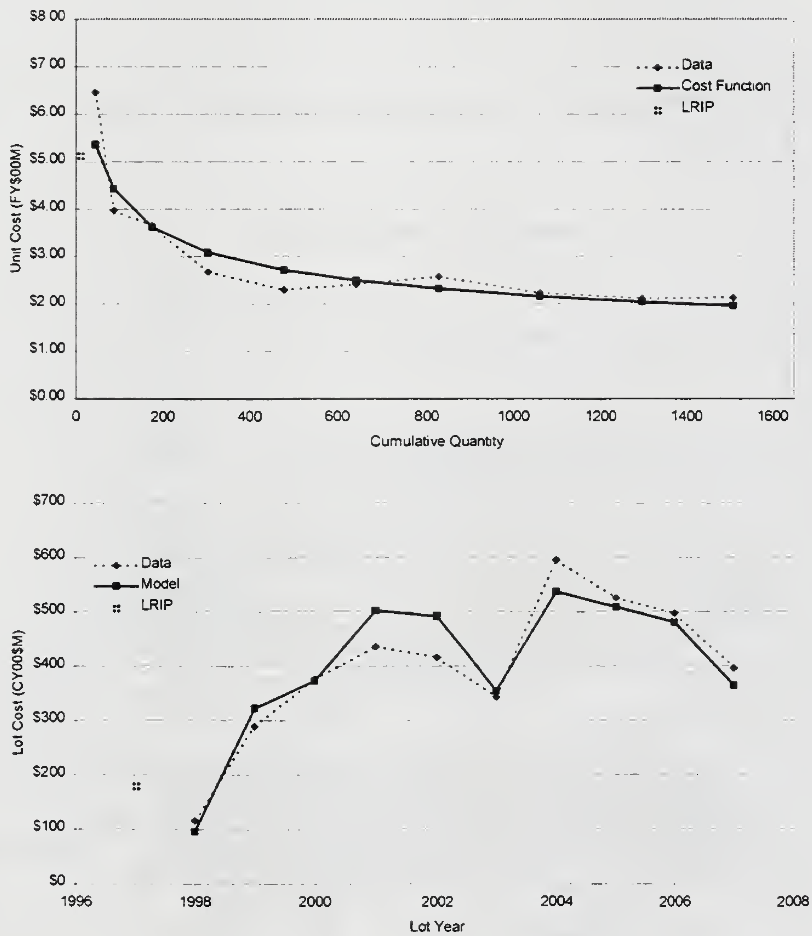


Figure 4. Base Modeling of Bradley Infantry Fighting Vehicle.
The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a R^2_{adj} of 0.88.

D. MULTIPLICATIVE RATE MODEL

1. Characteristics and Assumptions

Since the Base Model doesn't reflect an increase in total cost as production is extended over time, we extend it to include the *rate* of production. Lee describes this as the production feature most commonly added, with this form

$$C(Q) = T_1 Q^{\beta} R^{\chi} \tag{11}$$

where R denotes the number of units produced in a production period, and the constant χ is the rate exponent of the curve. The quantity 2^{χ} is the curve's "rate slope." As with β, χ is assumed to be negative. This implies that the unit cost of a system decreases as the rate at which it is produced increases. We refer to this model as the "Multiplicative Rate Model." Figure (5) shows this cost relationship for the Bradley Infantry Fighting Vehicle.

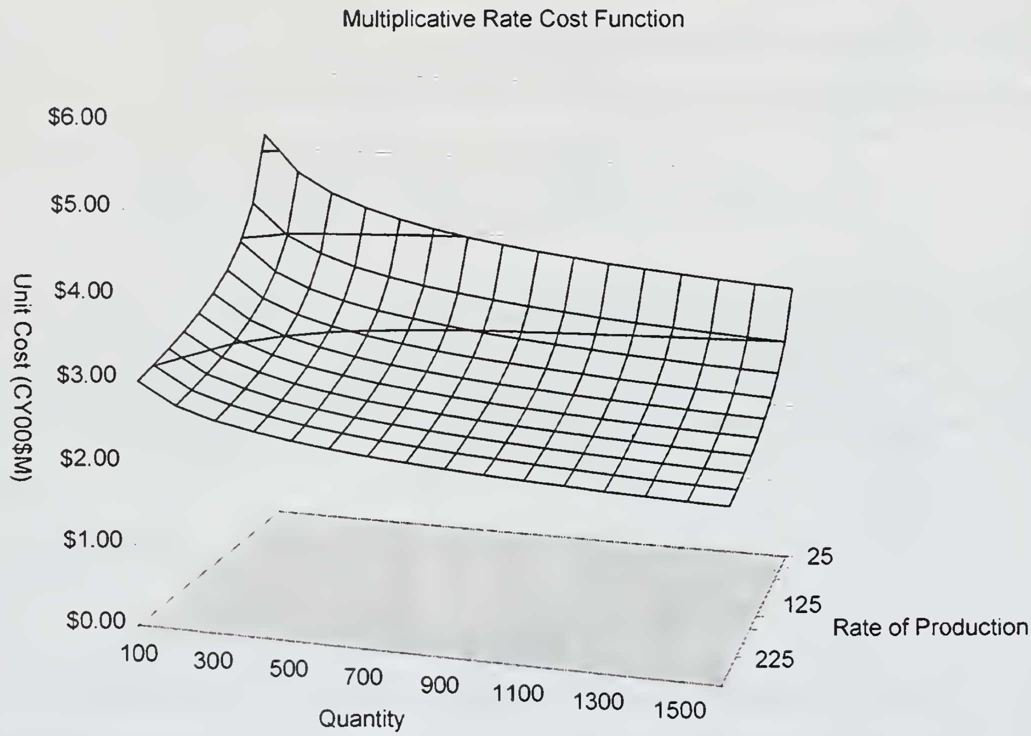


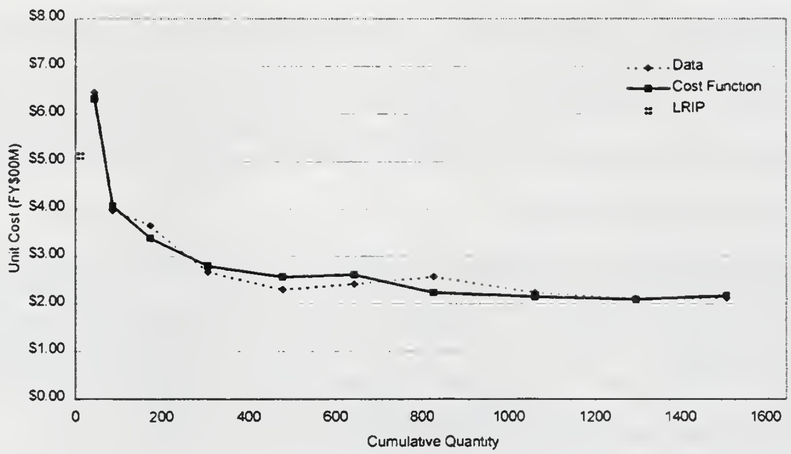
Figure 5. Multiplicative Rate Cost Function
In the Multiplicative Rate Model, both the quantity produced and the rate of production affect the unit cost. The chart shows this relationship for the Bradley Infantry Fighting Vehicle, where T_1 = 21.790; β = -0.136; and χ = -0.250

2. Model Fit

Fitting the Multiplicative Rate Model is done in a similar manner as with the Base Model, in that equation (11) is transformed to

$$\ln C(Q) = \ln T_1 + \beta \ln Q + \chi \ln R \tag{12}$$

and a linear regression performed to obtain the coefficients T_1, β, and χ. Results of the regressions for each of the systems are summarized in Appendix B. Predictably, the Multiplicative Rate Model more accurately fits the data, as compared to the Base Model. The median R^2_{adj} for this model over all systems is 0.88; the mean is 0.82. Parameters for each system and respective R^2_{adj} values are displayed in Appendix B. Figure (6) shows the data and fitted values for the BIFV unit and lot costs.



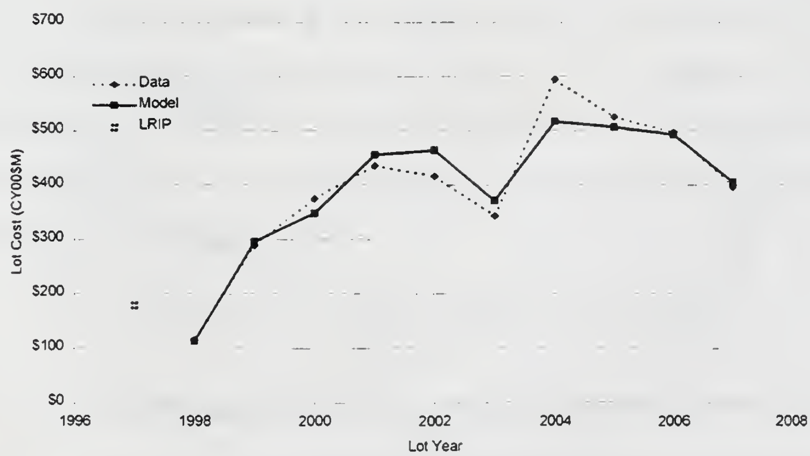


Figure 6. Multiplicative Rate Modeling of Bradley Infantry Fighting Vehicle
Bradley Infantry Fighting Vehicle plots of Data and Fitted Values. Estimated costs are determined by Multiplicative Rate Model. The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a R^2_{adj} of 0.91.

E. IDA RATE-PENALTY MODEL

1. Characteristics and Assumptions

The Multiplicative Rate Model possesses the important characteristic that as the number of years spent in production is extended, the total cost of a system increases. An issue for consideration regarding the data, however, is that if the rate of production increases through the production periods, rate and quantity can be highly correlated. IDA chose not to use this extension to the Base Model due to these perceived statistical problems (Balut, 1988). Instead, IDA developed their own cost function, of the form

$$C(Q) = T_1 Q^{\beta} + \delta \frac{\left(R^{*} - R\right)^2}{R^{*}} \tag{13}$$

where δ is the rate penalty parameter, R^* is the theoretically optimal rate of production for the system, and R is the actual rate of production for that period. This embellishment of the Base Model adds a penalty for production made at other than the optimal rate. This is referred to as the "IDA Model."

A useful feature of this relationship is that it does not assume that a higher production rate is necessarily better. This allows for the possibility of modeling the impact of paying overtime costs to increase production in a period, for example. Figure (7) shows the per unit and per lot penalties for the BIFV.

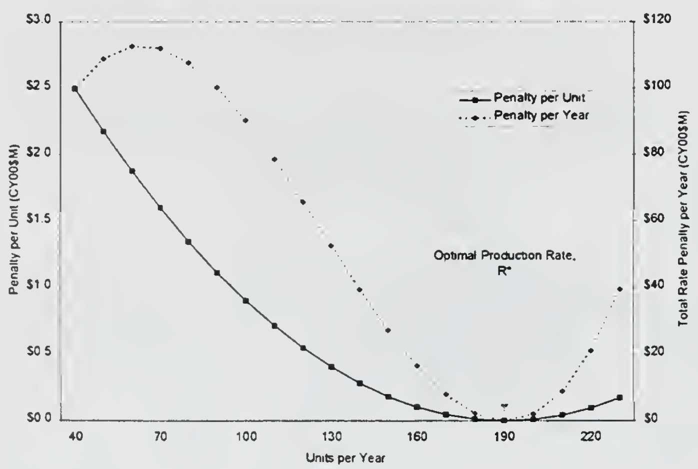


Figure 7. IDA Rate-Penalty Model Unit and Annual Penalties for Different Rates of Production for the Bradley Infantry Fighting Vehicle

The per unit penalty is $\delta(R^*-R)^2/R^*$, where: R^*, the theoretically optimal rate of production, is 190.55 per year; R is the actual rate of production; and δ, the penalty constant, is 0.021.

The per unit rate penalty in the IDA Model grows quadratically as the production rate departs from the optimal rate; that is, a deviation from the optimal production rate is penalized more if done in a single period than if divided over several periods. The predictable impact of this feature is that large deviations will be avoided. The optimal production schedule will tend to divide the total quantity of each system required evenly over the periods of procurement.

2. Model Fit

The IDA Model is more complicated to fit than either of the previous two developed thus far. The Excel solver to fit the function parameters in minimizing the sum of the squared differences between the data and the calculated average unit cost per period. The equation to calculate the average unit cost per period was derived as follows: the simple cost progress curve, equation (1), is integrated to yield the relationship between the total cost and cumulative quantity produced

$$T(Q) = \frac{T_1 Q^{\beta+1}}{\beta+1} \tag{14}$$

The SAR and DPP data provides the total cost for producing a given lot size. Therefore, the above equation is transformed to

$$TLC = \frac{T_1\left((Q_U + 0.5)^{\beta+1} - (Q_L + 0.5)^{\beta+1}\right)}{\beta+1} \tag{15}$$

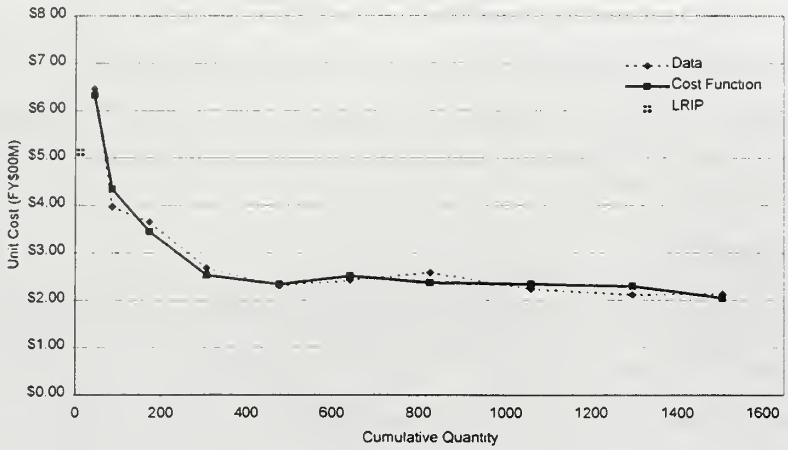
where TLC is the total lot cost, Q_U is the ending cumulative quantity once the lot is built, and Q_L is the ending cumulative quantity of the previous lot. Adding 0.5 to the beginning and ending quantities provides a better approximation to the continuous cost function.

Dividing equation (14) by R, the size of the lot, yields the average unit cost. To this the rate penalty is added as follows

$$AUC = \frac{T_1\left((Q_U + 0.5)^{\beta+1} - (Q_L + 0.5)^{\beta+1}\right)}{(\beta+1)R} + \delta\frac{(R^* - R)^2}{R^*} \tag{16}$$

where R^* is the theoretically optimal rate of production for that system, recognizing that R is equal to $(Q_U - Q_L)$. If R^* is not explicitly available from the manufacturer, it is either determined by the solver or set at the maximum production rate, depending on what the data suggested was most appropriate. The median R^2_{adj} for this model over all systems is 0.87; the mean is 0.77. Parameters for each system and respective R^2_{adj} values

are displayed in Appendix B. Figure (8) shows the data and fitted values for the BIFV unit and lot costs.



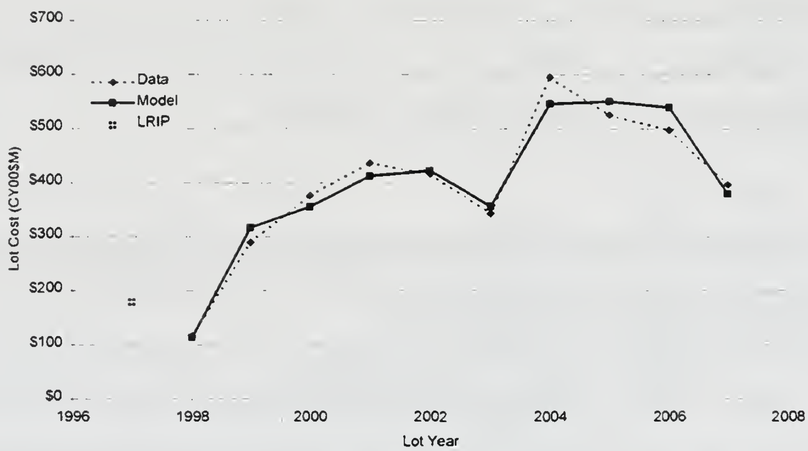


Figure 8. IDA Rate-Penalty Modeling of Bradley Infantry Fighting Vehicle
The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a R^2_{adj} of 0.94.

F. LINEARIZED-UNIT-RATE-PENALTY (LURP) MODEL

1. Characteristics and Assumptions

An assumption that the per unit penalty should more correctly be modeled as linear leads to a variation of the IDA Model of the form

$$C(Q) = T_1 Q^{\beta} + \delta\left|R^* - R\right| \tag{17}$$

Under this relationship, the same penalty is incurred for a deviation from the optimal production rate whether concentrated in one period or spread over several. Figure (9) shows the per unit and per lot penalties for the BIFV.

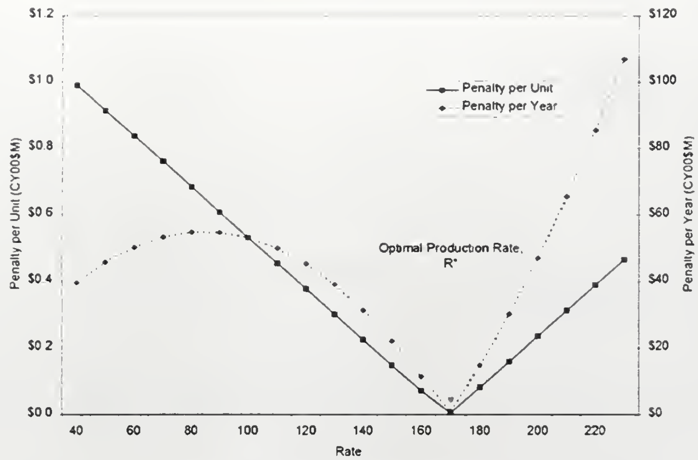


Figure 9. LURP Model Unit and Annual Penalties for Different Rates of Production for the Bradley Infantry Fighting Vehicle

Bradley Infantry Fighting Vehicle penalties per unit and per lot for different rates of production. The per unit penalty is $\delta|R^*-R|$, where: R^*, the theoretically optimal rate of production, is 169.27; R is the actual rate of production; and δ, the penalty constant, is 0.008.

2. Model Fit

Fitting this model is accomplished in the same manner as the IDA Model, except that the penalty term in equation (16) is replaced with the penalty term in (17). The median R^2_{adj} for this model over all systems is 0.84; the mean is 0.78. Parameters for each system and respective R^2_{adj} values are displayed in Appendix B. Figures (10) and (11) show the data and fitted values for the BIFV unit and lot costs.

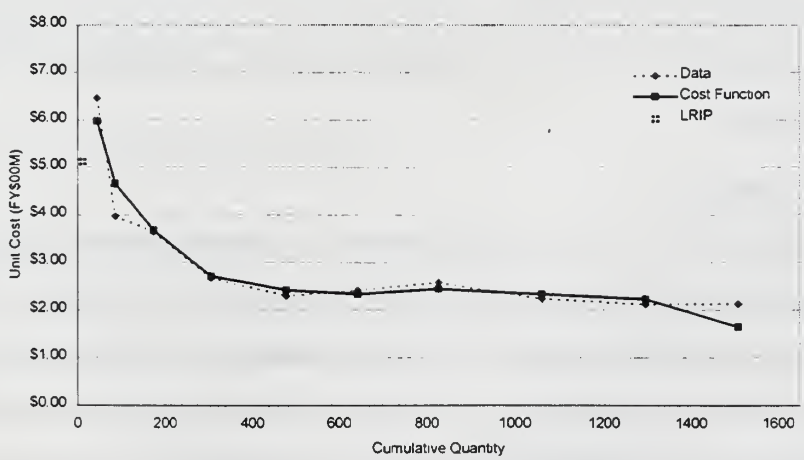


Figure 10. LURP Modeling of BIFV: Unit Costs

Bradley Infantry Fighting Vehicle plot of unit Data and Fitted Values. Estimated costs are determined by the LURP Model.

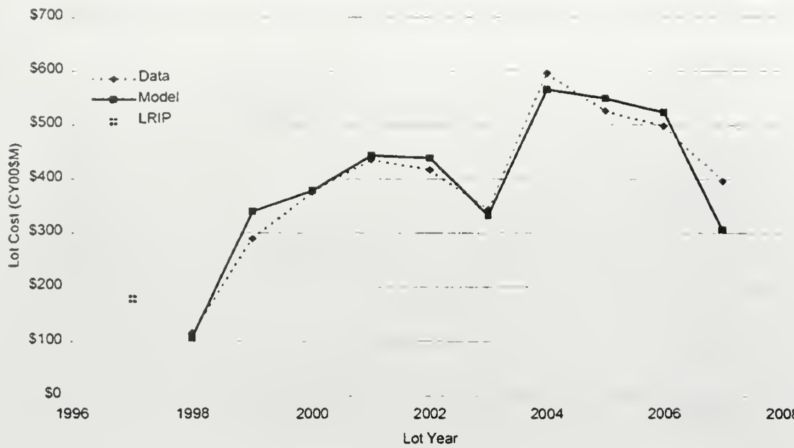


Figure 11. LURP Modeling of BIFV: Unit Costs
Bradley Infantry Fighting Vehicle plot of lot Data and Fitted Values. Estimated costs are determined by the LURP Model. The fitted values of the plot correspond to a R^2_{adj} of 0.88.

G. RATE-CHANGE MODEL

1. Characteristics and Assumptions

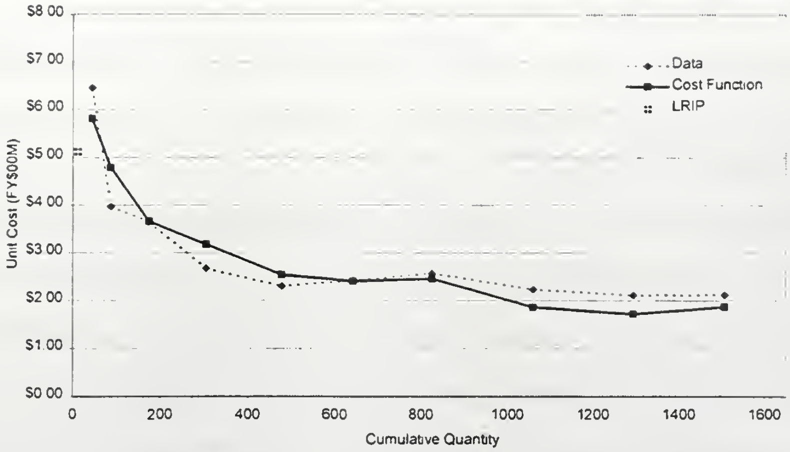
One can argue that a plant will optimize its production at any rate, given time to adapt. Changes in the rate of production between periods, however, prevent this goal from being realized. Another Base Model extension that attempts to capture this behavior is

$$C(Q) = T_1 Q^{\beta} + \delta|\Delta R| \tag{18}$$

where ΔR is the change in rate from one period to the next. We refer to this model as the "Rate-Change Model."

2. Model Fit

The parameters for the Rate-Change Model are determined in the same way as those of the rate-penalty models, except that the rate penalty term is replaced by the rate penalty term in equation (18). Values for δ are constrained to be greater than or equal to 0.001 to ensure that a minimum penalty is assessed for changing production rate. For the first production period, ΔR is defined to be zero. The median R^2_{adj} for this cost function over all systems is 0.73; the mean is 0.68. Parameters for each system and respective R^2_{adj} values are displayed in Appendix B. Figure (12) shows the data and fitted values for the BIFV unit and lot costs.



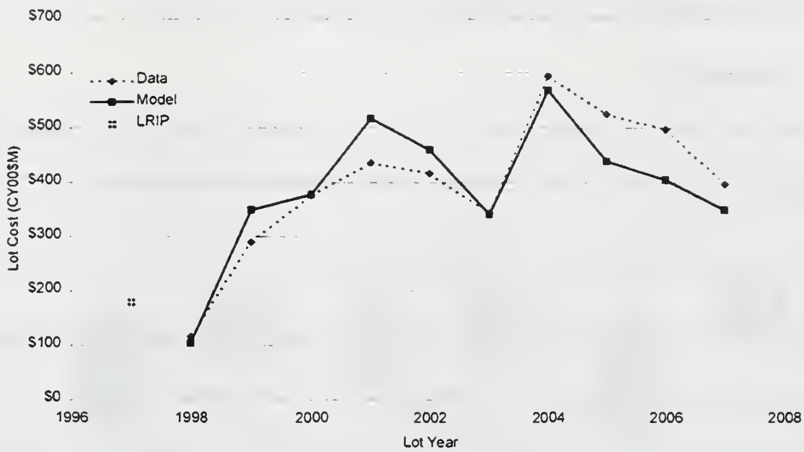


Figure 12. Rate Change Modeling of Bradley Infantry Fighting Vehicle
Bradley Infantry Fighting Vehicle plots of Data and Fitted Values. Estimated costs are determined by the Rate-Change Model. The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a R^2_{adj} of 0.79.

H. BASE + OVERHEAD MODEL

1. Characteristics and Assumptions

All of the modifications to the Base Model presented thus far, in some form, attempt to capture the effect of production rate. One can argue, however, that the rate effect is being confounded with the effect of overhead costs. A low production rate extends procurement over more periods, causing overhead costs to be incurred for a longer time. The simplest extension of the Base Model uses the learning curve to determine the learning portion of each lot cost, and then adds an overhead term to each

lot cost proportional to the production period. This cost function estimates lot costs directly, then, rather than unit costs. The form of this "Base + Overhead Model" is

$$TLC = \frac{T_1\left((Q_U)^{\beta+1} - (Q_L)^{\beta+1}\right)}{\beta+1} + \Omega t \tag{19}$$

where TLC is the total lot cost, Q_U is the ending cumulative quantity once the lot is built, Q_L is the ending cumulative quantity of the previous lot, Ω is the overhead term, and t is the fraction of the period in production. Since we cannot consider partial years of production, t is always one in our application of the model. Multiple optimal schedules that vary the production rate may be determined with this model, but they will all ensure that the system is procured in the shortest time possible.

2. Model Fit

Since the lot costs are fitted directly for this model, continuity corrections are added to equation (19), and the function fitted to the data is simply

$$TLC = \frac{T_1\left((Q_U+.5)^{\beta+1} - (Q_L+.5)^{\beta+1}\right)}{\beta+1} + \Omega \tag{20}$$

The Excel Solver is used as with the rate penalty models to determine the parameters for each system. The overhead term, Ω, is constrained to be greater than or equal to 1; this forces a minimum penalty for producing over an additional period. The median R^2_{adj} for this cost function over all systems is 0.90; the mean is 0.86. Parameters for each system and respective R^2_{adj} values are displayed in Appendix B. Figure (13) shows the data and fitted values for the BIFV unit and lot costs.

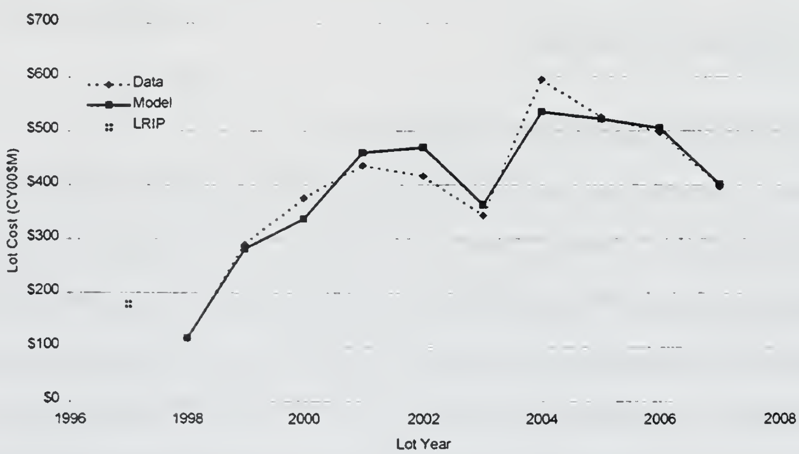


Figure 13. Base + Overhead Modeling of Bradley Infantry Fighting Vehicle
BIFV plots of Data and Fitted Values. The fitted values of the plot correspond to a R^2_{adj} of 0.93.

I. INITIAL COMPARISON OF MODELS

Using the median and mean R^2_{adj} for each cost function, the models are compared. Figure (14), a series of box plots for each model, summarizes the models into a useful visual tool for determining which is the most suitable. The most recently presented model, Base + Overhead, exhibits the highest median R^2_{adj} and also has the least variance. This can be considered the "best-fitting" model, followed, in order, by the Multiplicative Rate Model, the LURP Model, the IDA Rate-Penalty Model, the Base Model, and the Rate-Change Model. At this point, both the Rate-Change Model and the Base Model can be dropped from consideration for use as scheduling tools.

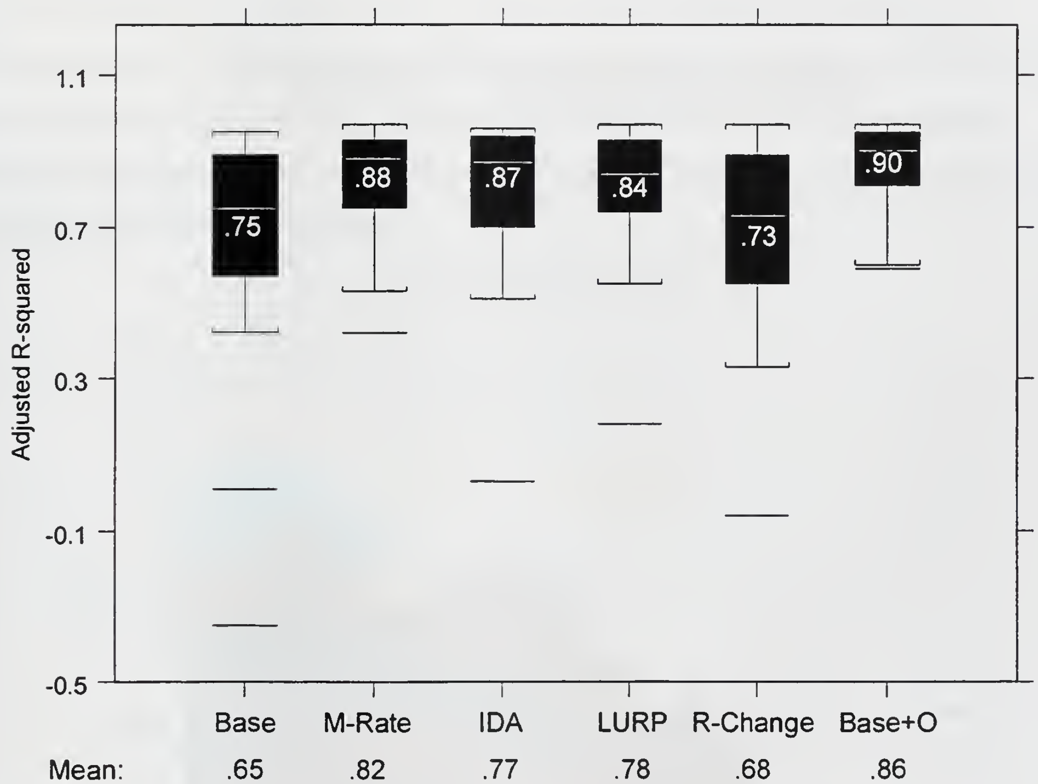


Figure 14. Box Plots of the Systems' R^2_{adj} Values for Each of the Cost Models Explored
Medians are labeled within the plots, means are labeled below the respective model name. The Base + Overhead model can be considered the "best-fitting" overall, as it yields the highest median R^2_{adj}, mean R^2_{adj} and the lowest variance.

THIS PAGE INTENTIONALLY LEFT BLANK

III. SPREADSHEET SCHEDULING TOOLS

A. INTRODUCTION

A worthwhile step before implementing a cost function in an optimizing program is to exercise it in a spreadsheet. In this manner we can confirm our beliefs as to how the function behaves, and perhaps gain further intuition as to how it will work in a math program. A strength of spreadsheets is that they can easily model complex relationships that are difficult, if not impossible, to implement in an optimization model. Elements of interest when employing our cost functions are: 1) how well the function estimates the current estimated total cost, given the current procurement schedule; 2) how the schedule "behaves" in moving towards optimality; and 3) to what degree the total cost is reduced when the schedule is "optimal." As a baseline, refer to Figure (15), a depiction of current projected acquisition costs derived from the data. The plot shows the total yearly cost if the seventeen systems are produced on currently anticipated schedules with currently anticipated costs. These costs will be called "true" since none of the estimation models of Chapter II are involved.

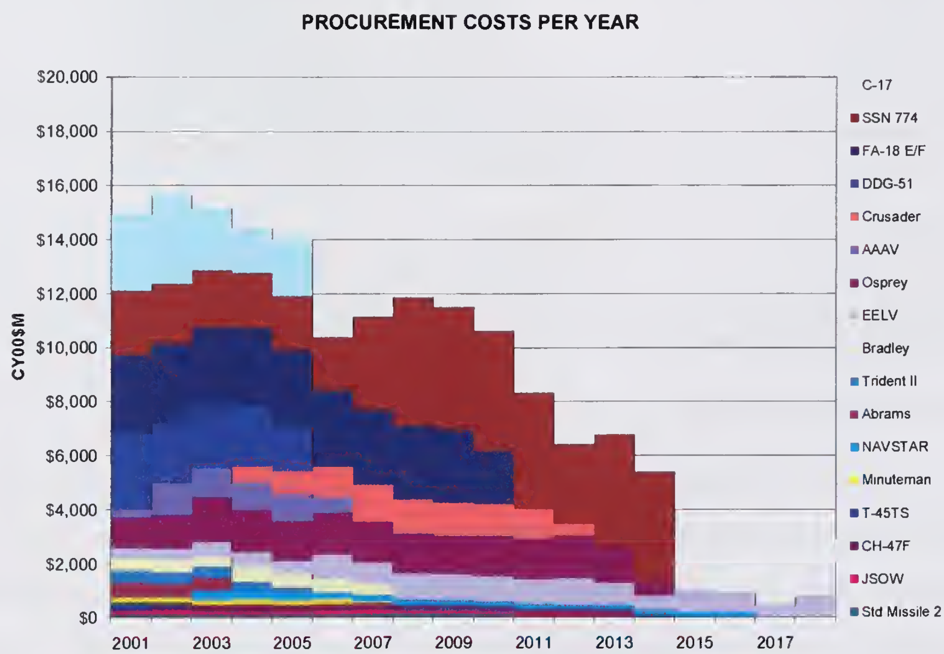


Figure 15. Cost of Current Procurement Schedule
Cost of current procurement schedule, by system, over eighteen year planning horizon (taken from data). The total cost of all systems will be $158,483 (CY00$M). The plot provides a baseline from which the effect of using the cost functions of interest can be readily observed.

As the spreadsheet model does not automatically enforce budget and production constraints, the user must ensure that they are observed. Because future needs are unknown, imposing a yearly constraint may not necessarily be wise; nonetheless, budget limits are a key feature of our model. In the following experiments with each cost function, annual budget constraints are relaxed while observing the production constraints. Bounding annual production rate constraints requires careful judgment by the modeler. If these limits are not provided, as in this situation, they must be extrapolated from the data available (Appendix A). For each system, the highest observed annual quantity procured is used as the maximum production rate. The minimum production rate is chosen much more subjectively; this constraint is essentially a control over production stability. Table (1) lists the maximum and minimum annual production rates used in the modeling efforts for each system.

System / Rate/Year	AAAV	Abrams	Bradley	C-17	CH-47F	Crusader
MAX	200	120	235	15	29	240
MIN	20	20	20	1	8	20
System / Rate/Year	**DDG-51**	**EELV**	**FA-18 E/F**	**JSOW**	**Minuteman III**	**NAVSTAR**
MAX	3	14	48	900	80	3
MIN	1	4	12	100	10	3
System / Rate/Year	**Osprey**	**SSN 774**	**Std Missile 2**	**T-45TS**	**Trident II**	
MAX	36	3	190	15	12	
MIN	9	1	60	1	5	

Table 1. Estimated Maximum and Minimum Annual Production Rates

B. MULTIPLICATIVE RATE MODEL

Using the Multiplicative Rate Model to determine the costs of the current procurement schedule yields a total cost of $153,241 (CY00$M). This is an underestimate of the true cost by 3.31%. A plot of the procurement costs as determined by the model is virtually indistinguishable from the data plot in Figure (14).

Experimentation confirms the expectation that as the rate of production increases, unit costs decrease. The minimum total cost is achieved when each system is produced at

maximum rate, until the quantity demanded is satisfied. Furthermore, it is most beneficial to delay any reduction in production as long as possible. Disregarding annual budget limits, the optimal procurement schedule simply produces every system at maximum rate, except during the last period. Figure (16) shows the least costly procurement schedule as determined using the Multiplicative Rate Model. The total estimated cost of this schedule is $146,489(CY00$M), a difference of $6,752(CY00$M) from the estimated current cost. Since this schedule did not consider annual budget limits, this difference serves as an upper bound on the savings that could be expected.

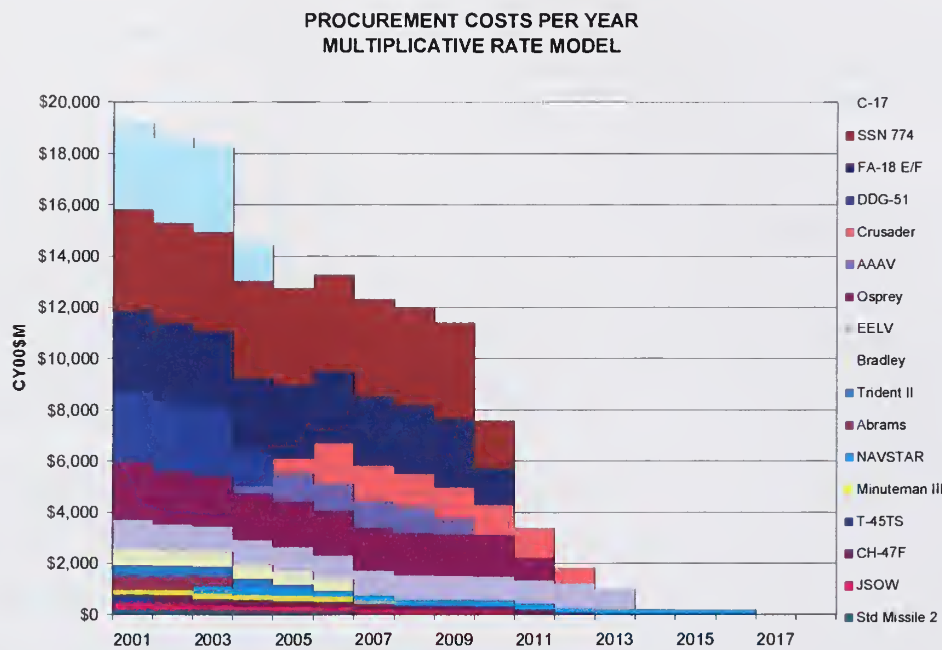


Figure 16. Cost of Optimal Procurement Schedule as Determined Using the Multiplicative Rate Model

The model schedules production at the maximum rate, until demand has been satisfied. The total cost of all systems is estimated to be $146,489 (CY00$M). This represents a savings of $6,752 (CY00$M) compared with the current schedule.

C. IDA RATE-PENALTY MODEL

The IDA Rate-Penalty Model yields a current-schedule total cost of $156,275 (CY00$M), an underestimate of the true cost by 1.39%. This is a slight improvement over the Multiplicative Rate Model. As mentioned earlier, when forced to produce at below optimum rate, this model seeks to divide the quantity required equally over the time allotted. Table (2) illustrates this property with a simple example.

R*= 100	PERIODS					Total Cost
δ = 10	1	2	3	4	5	
Current Rate	90	90	90	90	40	
Unit Penalty	10	10	10	10	360	
Lot Penalty	900	900	900	900	14400	18000
Current Rate	80	80	80	80	80	
Unit Penalty	40	40	40	40	40	
Lot Penalty	3200	3200	3200	3200	3200	16000

Table 2. IDA Rate-Penalty Model Unit and Annual Rate Penalties
Unit and lot rate-penalties for a five period hypothetical procurement schedule. The total quantity procured is 400. In this situation, the system is limited to producing 90 per period vice its optimum of 100. Producing at this maximum for four periods exposes the schedule to a severe penalty in the fifth period. The least costly solution divides the penalty evenly over the five periods.

An adjustment that is used in the spreadsheet implementation of this model is to eliminate the rate-penalty term of the cost function for the last period of production. Otherwise, as just discussed, the system would never be produced at the optimal rate, even when unconstrained by budget or rate limits. By eliminating the penalty for the last period, it is tacitly assumed that the system is produced at the optimal rate during a fraction of this period.

The optimal schedule, then, produces each system at its optimal rate, thus incurring no rate-penalty. The total cost of such a schedule will be the same as if it had been modeled using the Base Model. The total cost of this schedule is $148,824 (CY00$M), allowing an upper bound of $7,451 (CY00$M) in savings from the estimated cost of the current schedule. The plot of the IDA Rate-Penalty optimal schedule is nearly identical to that of the Multiplicative Rate Model shown in Figure (16).

D. LURP MODEL

The LURP Model yields a current-schedule total cost of $155,674 (CY00$M). This is an underestimate of the true cost by 1.77%. As with the IDA Rate-Penalty Model, the penalty term is dropped from the last production period. Unlike the IDA Rate-Penalty Model, this model seeks to produce at as close to optimal as possible, for as many periods as possible; table (3) illustrates this property.

R*= 100	PERIODS					Total Cost
δ = 10	1	2	3	4	5	
Current Rate	90	90	90	90	40	
Unit Penalty	100	100	100	100	600	
Lot Penalty	9000	9000	9000	9000	24000	60000
Current Rate	80	80	80	80	80	
Unit Penalty	200	200	200	200	200	
Lot Penalty	16000	16000	16000	16000	16000	80000

Table 3. LURP Model Unit and Annual Rate Penalties
Unit and lot rate-penalties for a five period hypothetical procurement schedule, calculated with the LURP Model. The total quantity procured is 400. In this situation, the system is limited to producing 90 per period vice its optimum of 100. Unlike the IDA Rate-Penalty Model, this model seeks to produce at as close to optimal as possible, for as many periods as possible. Dividing the penalty evenly over the five periods, the preferred solution for the IDA Rate-Penalty Model, is more costly.

Producing at the optimal rate for each system and period, without regard to budget limits, provides the minimum achievable total cost. The plot of the LURP optimal schedule is also nearly identical to that of the Multiplicative Rate Model shown in Figure (16). The total estimated cost of this schedule is $143,965(CY00$M), a difference of $11,709(CY00$M) from the current cost estimated with the same function. Again, since this schedule did not consider annual budget limits, this difference serves as an upper bound on the savings that could be achieved.

E. BASE+OVERHEAD MODEL

The Base + Overhead Model yields a total cost of $159,683 (CY00$M). This is an overestimate of the true cost by only 0.76%, the closest of the four functions modeled with spreadsheets. This is also the simplest of the four functions. The same overhead cost is included in the lot cost if any units are procured, regardless of the quantity, including the last production period. A tacit assumption is made that the final production period incurs the same overhead as a full period. There is no unique optimal solution to this scheduling exercise, since any combination of quantities that completes production in the shortest amount of time will incur the same cost.

Figure (17) illustrates one optimal solution determined using this function. The total procurement cost over this schedule is $155,407 (CY00$M). This is only $4,276 (CY00$M) less than the estimated cost of the current schedule.

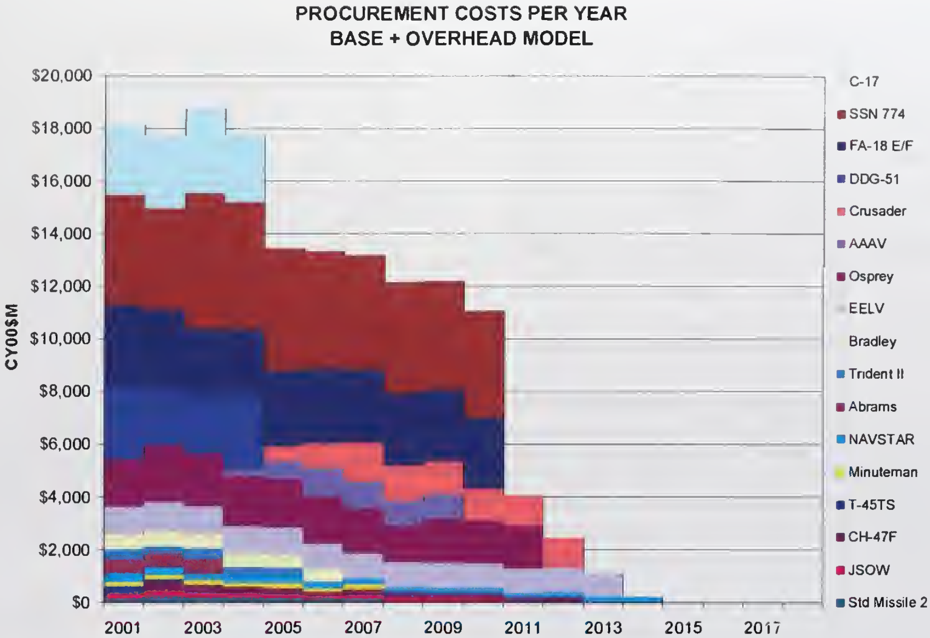


Figure 17. Cost of Optimal Procurement Schedule, Determined with the Base + Overhead Model

The same overhead cost is included in the lot cost if any units are procured, regardless of the quantity, including the last production period. There is no unique optimal solution to this scheduling exercise, since any combination of quantities that completes production in the shortest amount of time will incur the same cost. The total cost of all systems is estimated to be $155,407 (CY00$M). This represents a savings of $4,276 (CY00$M) compared with the current schedule.

F. FURTHER COMPARISON OF MODELS

Of the four functions implemented in spreadsheet applications, only the Base + Overhead Model overestimated the total cost of the current procurement schedule. The 0.76% error of this function's estimate is the smallest of the group in absolute value. Additionally, the potential savings from optimal scheduling is the lowest. It is reasonable to argue that the Base + Overhead Model is the most conservative of the four. Table (3) summarizes the statistics of interest for each of these models.

Model	Multiplicative Rate	IDA Rate Penalty	LURP	Base + Overhead
Median R^2_{adj}	.88	.87	.84	.90
Mean R^2_{adj}	.82	.77	.78	.86
Cur. Sched Est Cost	153,241	156,275	155,674	159,683
|Error|	3.31%	1.39%	1.77%	.76%

Table 3. Summary of Measures of Effectiveness of the Four Best Cost Models

G. "OPTIMAL" SPREADSHEET SOLUTION

The most accurate spreadsheet tool is selected based on the Base + Overhead cost function, for schedule planning. Using this tool, one can attempt to determine the optimal procurement schedule subject to any annual budget constraint. An estimated budget amount that will serve as a basis for later comparison, derived simply by estimating the largest annual cost from Figure (15), is $16,000 (CY00$M). Figure (18) shows the least costly schedule that the author was able to derive after several hours of experimentation and manipulation. The total cost of all systems on this schedule is estimated to be $159,159 (CY00$M), only $524 (CY00$M) less costly than the current schedule. Although the spreadsheet planner is useful for visualizing procurement costs, using it to find an optimal schedule is impractical and tedious.

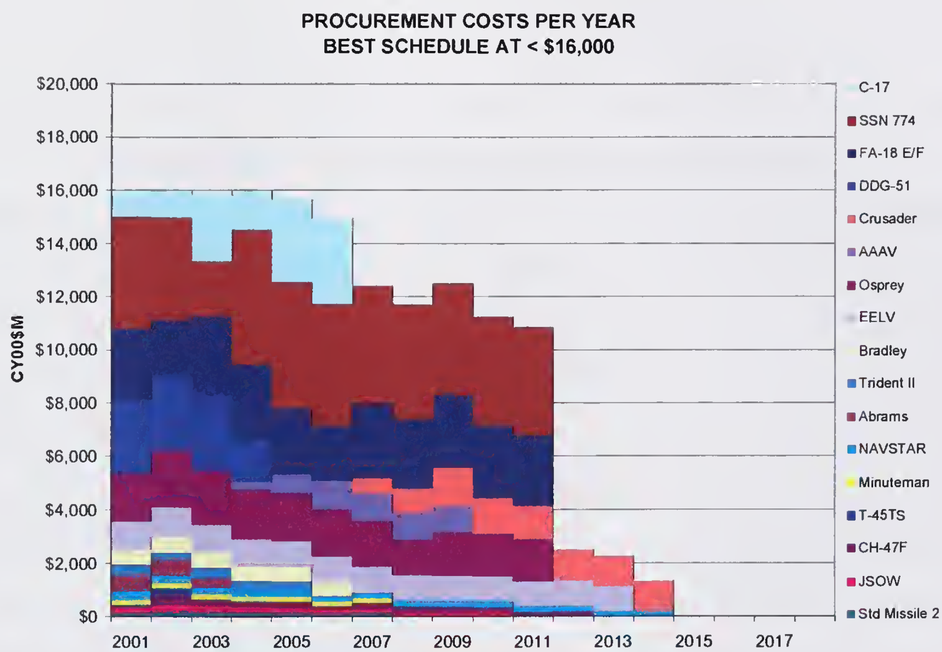


Figure 18. Schedule "Optimized" using Spreadsheet Planning Tool
The least costly schedule that the author was able to derive after several hours of experimentation and manipulation. The unconstrained optimal solution is used as a starting point, and lot quantities of various systems are changed until annual budget constraints are met. The total cost of all systems on this schedule is $159,159 (CY00$M), which is only $524 (CY00$M) less costly than the current schedule.

THIS PAGE INTENTIONALLY LEFT BLANK

IV. OPTIMIZATION TOOL

The time required in manipulating the spreadsheet tool, coupled with the lack of improvement over the current schedule, reinforce the belief that a truly optimizing planning tool is required. All of the cost functions developed herein are inherently nonlinear, however, a hindrance to optimization modeling. Fortunately, in addition to being the best fitting of the cost functions explored, the Base + Overhead cost function is the simplest to approximate linearly. This cost function is implemented in an optimization model, henceforth called the Procurement Scheduling Optimization Model (PSOM).

A. LINEARIZING THE COST FUNCTION

The nonlinear portion of the Base + Overhead cost function, equation (19), is the learning effect term, which is linearized using a step function. This is accomplished for each system, using the following steps: 1) the cost of each unit is determined using the Base Model; 2) the mean cost of the units *yet to be procured* is determined; 3) this set of units is divided into two subsets: the first consisting of all units of *above* average cost; the second consisting of units of *below* average cost; 4) both of these subsets are further divided into two groups in the same manner, yielding a total of four groups; and, lastly, 5) the mean unit cost of each group is assigned as the price for every element in that group. This method of dividing the set of items to be acquired attempts to weigh the size of each group appropriately, contributing to a good deal less error than the standard technique of dividing them into groups of equal number. Although somewhat more of a demanding procedure, the task is easily accomplished in a spreadsheet. Figure (19) illustrates the process described above. Figure (20) shows the cumulative cost curve of a system priced using the Base Model versus the same system using a step function approximation. Implementing the step function approximation into a spreadsheet computes a cost of $160,546 for the current procurement schedule. This is a represents a slight increase in error, which is to be expected, an overestimate of the true cost by 1.30%.

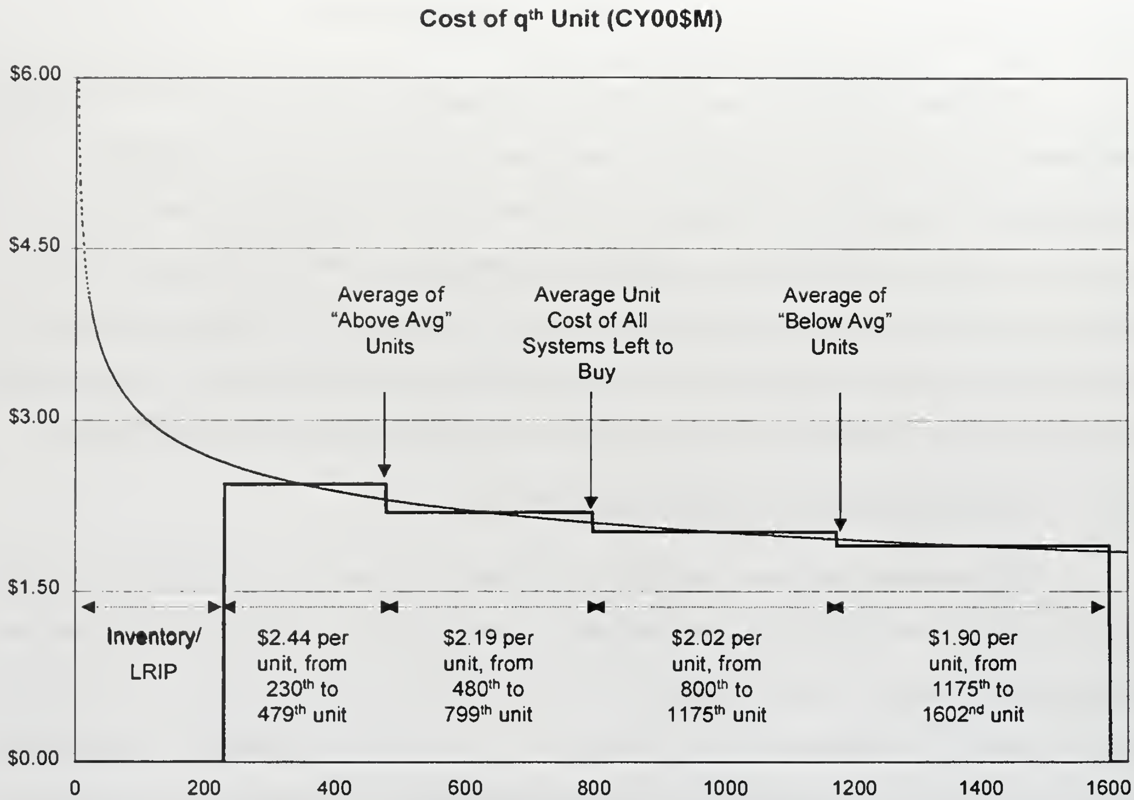


Figure 19. Linearization of the Learning Curve Using a Step Function

The average unit cost of the systems yet to be procured is determined. The set of units to be procured is divided into two subsets: the first with units of *above* average cost, the second with units *below* average cost. These subsets are further divided in the same manner as the original set, yielding a total of four groups. The average unit cost of each group is assigned to every item in that group.

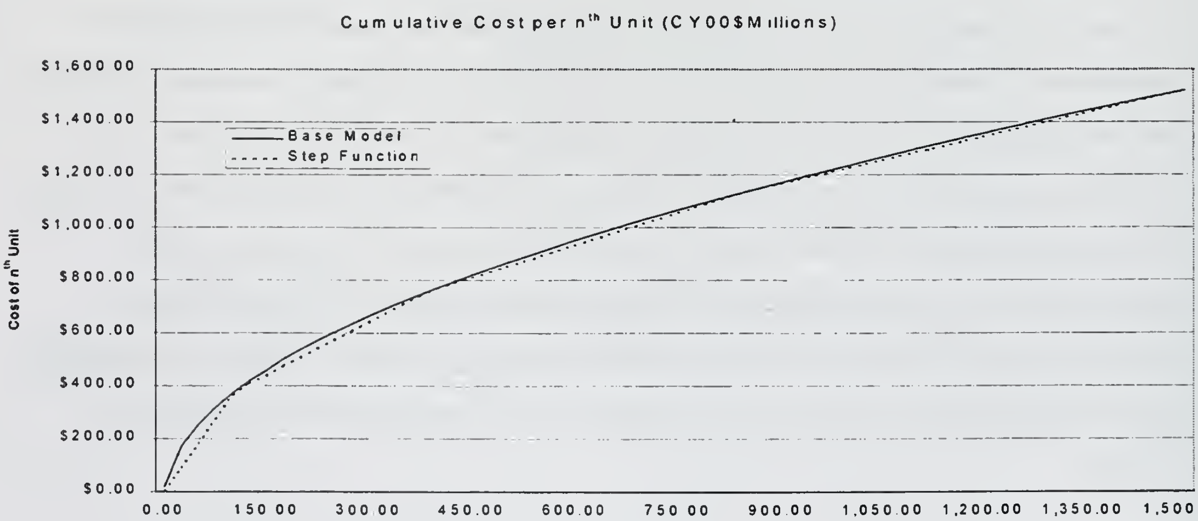


Figure 20. Cumulative Cost of a System: Base Model vs. Step Function

Comparison of cumulative system cost as determined by the Base Model and its step function approximation. The system shown has a T_1 of 20 and learning curve slope, exaggerated for clarity, of 71% (β = -0.5).

B. MODEL FORMULATION

1. Introduction

PSOM is an integer-linear program that recommends the quantity of each MDAP system to be procured in each year of the DPP. PSOM carries the same assumptions as the Base + Overhead cost which it employs; the most significant of these is that periods are measured in years, with partial periods of production not allowed. Thus, the overhead incurred when a system is in production is the same for each year.

The model uses both binary and continuous variables to achieve a balance of realism and solvability. Binary variables are used to indicate the start periods and periods of full rate production (FRP) for each system. Additional binary variables are used to control the step function approximation of the cost function. Although procurement quantities must be integers in reality, they are represented by continuous variables to expedite solution.

2. Formulation

<u>Indices</u>

i	system
t, t'	time period $\{1 \ldots N\}$
p	LRIP period $\{1 \ldots M_i\}$
c	cost level $\{1 \ldots 4\}$

<u>Data</u>

N	number of periods under consideration in this model
M_i	number of LRIP periods for system i
$QLRIP_{ip}$	number of units i in LRIP period p
$CLRIP_{ip}$	procurement cost of system i in LRIP period p
$PRICE_{ic}$	unit cost of system i at cost level c
$OVERHEADi$	fixed cost incurred by system i if in FRP
$MINBUY_{ic}$	quantity of system i that must procured at cost level c before they may be procured at cost level $c + 1$
$BUDGET_t$	annual budget constraint

$DEMAND_{it}$	required minimum quantities of system i by period t
INV_i	starting inventory of system i in period 1
$MINRATE_i$	minimum sustaining production rate for system i during FRP
$MAXRATE_i$	maximum production rate for system i during FRP
$EARLY_i$	earliest period in which FRP may begin for system i ($> M_i$)
$LATE_i$	latest period in which FRP may begin for system i ($\leq N$)

Variables

q_{it}	units of system i procured in period t
$start_{it}$	{1 if system i begins FRP in period t, else 0
frp_{it}	{1 if system i is in FRP in period t, else 0
y_{itc}	{1 if system i is available at price level c in period t, else 0
$cumq_{it}$	cumulative units of system i procured by period t
$cost_{it}$	cost of all units of system i procured in period t

Formulation

MINIMIZE: $\sum_i \sum_t cost_{it}$

SUBJECT TO:

Budget

$$cost_{it} = \sum_c q_{itc} PRICE_{ic} + frp_{it} OVERHEAD_i$$

$$+ \sum_i \sum_{p=1}^{M_i} CLRIP_{ip} \left(start_{i(1+t+M_i-p)} \right) \quad \forall i,t \tag{C1}$$

$$BUDGET \geq \sum_i cost_{it} \quad \forall t \tag{C2}$$

Demand

$$cumq_{it} = INV_i + \sum_{t'=1}^{t} \sum_c q_{it'c} + \sum_{t'=1}^{t} \sum_{p=1}^{M_i} QLRIP_{ip} \left(start_{i(1+t'+M_i-p)} \right) \quad \forall i,t \tag{C3}$$

$$cumq_{it} \geq DEMAND_{it} \quad \forall i,t \tag{C4}$$

Production Rates

$$\sum_{c} q_{itc} \geq frp_{it} MINRATE_i \quad \forall i,t \tag{C5}$$

$$\sum_{c} q_{it} \leq frp_{it} MAXRATE_i \quad \forall i,t \tag{C6}$$

FRP Start

$$\sum_{t} t\left(start_{it}\right) \geq EARLY_i \quad \forall i \tag{C7}$$

$$\sum_{t} t\left(start_{it}\right) \leq LATE_i \quad \forall i \tag{C8}$$

No Production Breaks

$$\sum_{t} start_{it} = 1 \quad \forall i \tag{C9}$$

FRP Start/Production Status Integrity

$$start_{it} \geq \left(frp_{it} - frp_{it-1}\right) \quad \forall it \tag{C10}$$

Step Function Approximation and Buy Limit per Cost Level

$$y_{it1} MINBUY_{i1} \leq \sum_{t'=1}^{t} q_{it1} \leq MINBUY_{i1} \quad \forall it \tag{C11}$$

$$y_{it2} MINBUY_{i2} \leq \sum_{t'=1}^{t} q_{it2} \leq MINBUY_{i2} y_{it1} \quad \forall it \tag{C12}$$

$$y_{it3} MINBUY_{i3} \leq \sum_{t'=1}^{t} q_{it3} \leq MINBUY_{i3} y_{it2} \quad \forall it \tag{C13}$$

$$y_{it4} MINBUY_{i4} \leq \sum_{t'=1}^{t} q_{it4} \leq MINBUY_{i4} y_{it3} \quad \forall it \tag{C14}$$

Variables

$$q_{itc}, cumq_{it}, cost_{it} \geq 0 \quad \forall i,t \tag{C15}$$

$$frp_{it}, start_{it}, y_{itc} \in \{0,1\} \quad \forall i,t \tag{C16}$$

3. Explanation of Constraints

Constraints (C1) define the cost of all units of system i procured in period t, $cost_{it}$. The most complicated term in these constraints is the "LRIP term"

$$\sum_{i}\sum_{p=1}^{M_i} CLRIP_{ip}\left(start_{i(1+t+M_i-p)}\right) \quad \forall i,t$$

which is the LRIP costs in time period t due to LRIP for system i. A hypothetical system with three LRIP periods, a first LRIP period cost of X, and FRP beginning in period six provides an example (we omit the i subscript for brevity):

$M = 3$; $CLRIP_1 = X$; $start_6 = 1$

we see that in time period $t = 3$, the LRIP cost is

$$CLRIP_1\left(start_{(1+t+M_i-p)}\right) = X\left(start_{(1+3+3-1)}\right) = X\left(start_6\right) = X * 1 = X$$

as is required. Similar results are obtained for LRIP periods 2 and 3 during time periods 4 and 5 respectively.

Constraints (C2) ensure that costs per year never exceed the annual budget. Constraints (C3) define the cumulative quantity of each system procured by period t, and Constraints (C4) ensure that this quantity meets demand. Constraints (C5) and (C6) ensure that annual rates of production are maintained between their respective upper and lower limits. Constraints (C7) and (C8) imposed a window of allowable periods in which FRP may begin ($\sum_t t\left(start_{it}\right)$ is simply the FRP start period for system i.) Constraints (C9) make certain that there are no breaks in production. Constraints (C10) maintain consistency between the indicator variables for $start_{it}$ and frp_{it}. Constraints (C11) through (C14) enforce the step function approximation to the learning curve. The binary variables y_{ic} ensure that all of the more expensive units are procured in the required quantity before allowing the next less expensive units to be procured. This is the same "toggling" technique used by Loerch (1997).

C. IMPLEMENTATION AND ANALYSIS

1. Implementation

PSOM is implemented in the General Algebraic Modeling System (GAMS) [Brooke et al. 1997] with the CPLEX solver, Version 6.5 [ILOG 1999]. Over an 18-year planning horizon seventeen MDAP systems are scheduled, at four cost levels. The model

has approximately 11,000 equations and 3,500 variables, of which 1,600 are binary. Appendix C is the GAMS implementation of PSOM.

PSOM is a mixed-integer linear program, and is solved by branch-and-bound enumeration. The time required to solve each run of the model is influenced by the relative integer termination tolerance; this is the difference between the best integer solution and the best known lower bound, divided by the absolute value of the best integer solution. With a relative tolerance of 0.001, PSOM generally runs in less than seven minutes on a personal computer equipped with a Pentium II 333MHZ processor and 296MB of RAM. With a relative tolerance of 0.0005, the mean time to solve is increased to 15 minutes. While a relative tolerance of one tenth of one percent certainly represents a higher fidelity and resolution than the underlying input data, we used a 0.0005 relative tolerance to more accurately compare model results at varying budget levels.

2. Computational Results

Since the seventeen systems modeled by PSOM are only a subset of the eighty-plus systems that comprise the MDAP population, there is no accurate budget amount to input to the model. We use the annual budget amount of $16,000 (CY00$M) derived earlier and used in our spreadsheet "optimization." At this budget level, PSOM determines the optimal schedule depicted in Figure (21). PSOM minimizes cost by eliminating as much overhead as possible. Total cost of this schedule, as determined by GAMS, is $156,297 (CY00$M); this is 0.26% more than the spreadsheet-calculated cost of the same schedule, which we attribute to rounding and linearization error. Comparing the cost of the current to the optimal schedule, a potential savings of $3,792 (CY00$M) is revealed. Additionally, we notice that this is $3,268 (CY00$M) less than the least costly spreadsheet-determined schedule.

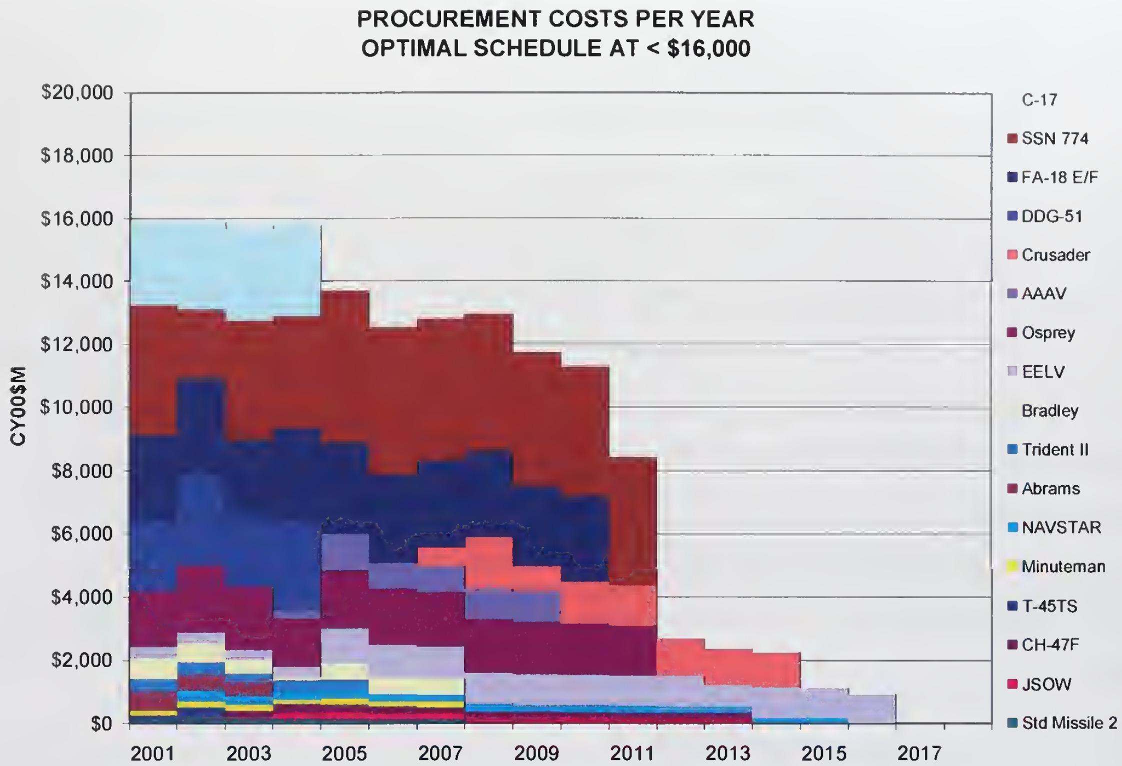


Figure 21. Optimal Schedule at Current Budget Level
The optimized procurement schedule at the current estimated annual budget limit. The optimized schedule represents a potential savings of $3,792 (CY00$M) over the current schedule.

PSOM can be used to construct a chart of the efficiency frontier, a plot of the minimum total cost of all systems at varying budget limits. This is built by repeatedly solving PSOM in a loop, with the budget decreasing after each iteration, from an amount in which the constraint is slack to the point at which the model becomes infeasible. A simple modification to PSOM, in which the budget is minimized vice the total cost of all systems, reveals the absolute minimum annual budget to be $12,714 (CY00$M). Figure (22) is the efficiency frontier for this procurement schedule. Procurement schedules corresponding to all points above the line are sub-optimal. Schedules corresponding to points below the line are infeasible. The decision maker can use the information in many ways. Assuming that the current schedule is sub-optimal, the decision maker may choose to optimize the schedule for the current budget and thus reduce overall cost; or, given an allowable overall cost, the decision maker may choose to reduce the annual budget available. If the schedule is already optimal, the decision maker will readily appreciate

additional cost of a reduction in budget, or the potential long-term savings from an increased budget, are equally apparent.

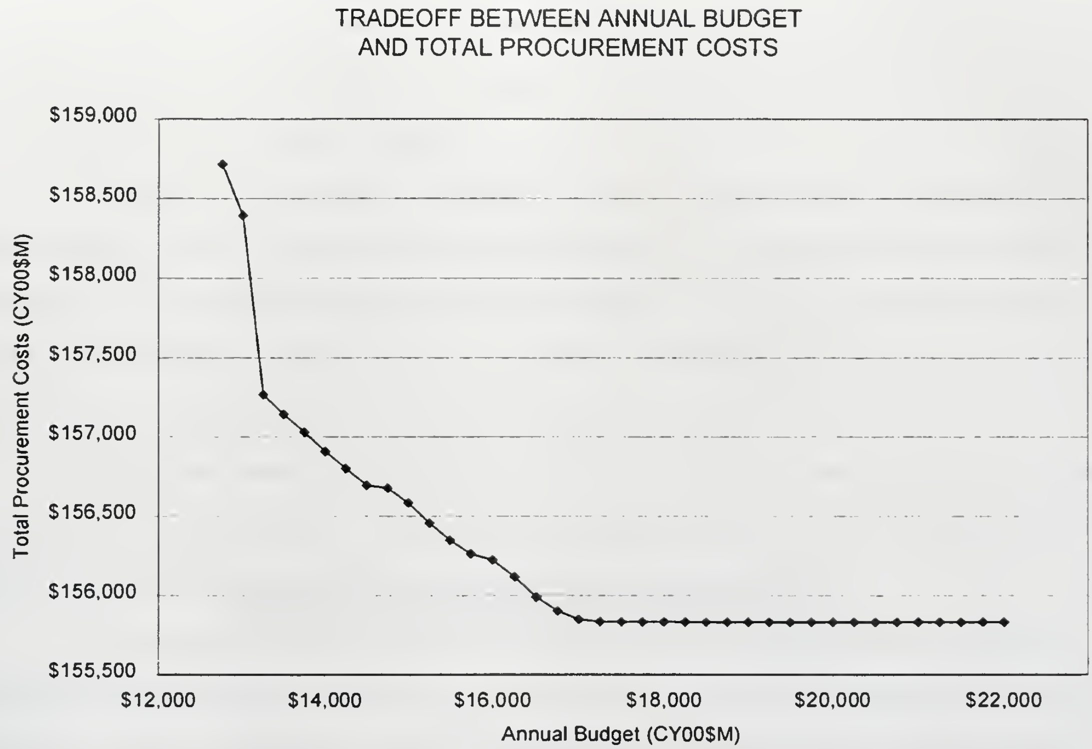


Figure 22. Efficiency Frontier for MDAP Procurement Schedule
Procurement schedules corresponding to all points above the line are sub-optimal. Schedules corresponding to points below the line are infeasible. Assuming that the current schedule is sub-optimal, the decision maker may choose to optimize the schedule for the current budget and thus reduce overall cost; or, given an allowable overall cost, the decision maker may choose to reduce the annual budget available. If the schedule is already optimal, the decision maker will readily appreciate the affect of changing the annual budget limit; the additional cost of a reduction in budget, or the potential long-term savings from an increased budget, are equally apparent.

THIS PAGE INTENTIONALLY LEFT BLANK

V. CONCLUSION

Of the six cost functions derived from the Unit Theory Model (equation 1), the Base + Overhead Model is most suited to this purpose; it shows the best fit to the data, most accurate estimate of current total cost, and is the simplest to implement in an integer linear program. PSOM uses this relationship to determine the annual procurement costs of the MDAP systems that it schedules.

PSOM allows the analyst to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest FRP start periods; and LRIP costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints. Data input to the model requires a working knowledge of GAMS.

Perhaps the most illuminating product that PSOM can determine is the efficiency frontier for the schedule cost versus the annual budget limit. This allows a decision maker to visualize the tradeoff between total and annual costs.

PSOM can be easily expanded to include all 80+ MDAP systems, with some slowing of solution time. Solution times can be reduced by manipulating the model's relative termination tolerance, while maintaining the required fidelity of the model solution. Thus, PSOM is a useful tool available to acquisition planners and decision makers. Expansion and use of PSOM or a similar optimization model is recommended for the upcoming and subsequent QDRs.

THIS PAGE INTENTIONALLY LEFT BLANK

APPENDIX A: DATA

	AAAV		Abrams		Bradley		C-17		CH-47F		Crusader	
YEAR	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost
1985												
1986												
1987												
1988							2	1018.93				
1989							4	1531.84				
1990							4	1865.50				
1991			62	751.33			4	1980.06				
1992			0	0.00			6	2119.84				
1993			0	0.00			6	2349.68				
1994			172	578.58			6	2490.52				
1995			34	313.38			8	2659.00				
1996			100	597.03			8	2128.38				
1997			120	491.66	35	179.16	9	2304.65				
1998			120	609.40	18	116.20	13	3015.49				
1999			120	693.39	73	289.83	15	3343.07				
2000			120	649.22	103	375.99	12	2835.92				
2001			105	539.52	163	436.08	15	3138.41				
2002			90	579.01	181	416.41	9	2278.40	11	166.28		
2003			88	515.32	142	343.63	5	1378.91	17	201.25		
2004					231	595.57	8	2032.88	27	275.20	62	583.00
2005	38	249.40			235	525.54			29	266.80	114	911.80
2006	200	1194.50			235	497.49			26	208.92	180	1259.00
2007	200	1004.40			186	396.09			26	204.09	240	1280.00
2008	200	950.40							26	200.73	240	1276.00
2009	200	926.60							26	197.79	240	1176.00
2010	112	636.70							26	194.95	240	1168.00
2011									26	193.06	240	1168.00
2012									26	190.75	74	452.00
2013									26	148.73		
2014									8	46.95		
2015												
2016												
2017												
2018												

Table A1. Data for AAAV, Abrams, Bradley, C-17, CH-47F, and Crusader
All costs CY00$M

	DDG-51		EELV		FA-18 E/F		JSOW		Minuteman		NAVSTAR	
YEAR	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost
1985	1	1582.53										
1986	3	3029.90										
1987	4	3328.72										
1988	5	4053.61										
1989	4	3407.59										
1990	5	4224.52										
1991	4	3544.24										
1992	3	2930.33										
1993	3	2867.45										
1994	2	2170.10										
1995	4	3479.48										
1996	4	3655.36							4	10.35		
1997	3	2823.24			12	2125.34	100	66.03	10	64.71		
1998	3	2769.90			20	2165.90	135	76.17	30	106.12		
1999	3	2846.09			30	2844.57	328	116.49	39	105.67		
2000	2	1977.56	1	67.60	36	2821.15	454	111.18	65	183.76		
2001	2	2038.16	4	351.25	42	2913.62	636	161.40	80	190.29		
2002	2	2009.54	5	430.90	45	2848.91	747	170.09	80	180.95		
2003	1	1276.99	7	512.93	48	2912.05	709	162.00	80	190.52	3	329.80
2004			7	476.04	48	2923.52	603	128.56	80	181.29	3	286.07
2005			6	393.24	48	2937.16	504	99.59	80	178.93	3	243.57
2006			12	1121.46	48	2835.76	893	163.21	80	182.08	3	237.03
2007			11	769.02	48	2756.44	981	183.97	24	106.91	3	223.55
2008			13	1031.83	48	2720.23	675	97.42			3	289.75
2009			13	877.10	48	2666.99	675	96.81			3	303.03
2010			13	878.07	27	1703.55	675	103.57			3	229.68
2011			13	877.10			675	114.20			3	208.63
2012			14	861.47			675	113.35			3	194.73
2013			12	826.96			535	108.89			3	191.06
2014			8	522.48							3	181.86
2015			12	825.99							3	182.47
2016			11	686.55							3	182.68
2017			7	460.52								
2018			12	837.81								

Table A2. Data for DDG-51, EELV, FA-18 E/F, JSOW, Minuteman III, and NAVSTAR
All Costs CY00$M

	Osprey		SSN 774		Std Missile 2		T45TS		Trident II	
YEAR	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost	Qty	Cost
1985										
1986										
1987										
1988										
1989										
1990										
1991										
1992										
1993										
1994										
1995										
1996									6	497.24
1997	5	715.38			80	103.39			7	305.56
1998	7	700.64	1	2758.35	68	112.70	12	522.26	5	256.03
1999	7	676.46	1	2047.82	71	101.47	24	861.58	5	297.45
2000	11	955.27	1	1676.50	75	115.36	12	398.56	12	452.45
2001	16	1256.88	1	1888.31	75	112.11	12	305.68	12	423.10
2002	19	1460.36	1	1842.41	80	113.29	12	343.11	12	424.17
2003	28	1646.74	1	1837.09	88	105.02	12	279.09	12	395.13
2004	28	1579.61	1	2172.61	90	92.76	12	332.91	12	365.32
2005	28	1487.32	1	2756.18	90	85.37	12	308.71	5	451.99
2006	30	1480.37	2	3997.55	120	100.00	15	302.20		
2007	30	1428.67	3	4928.79	150	115.95	15	314.79		
2008	30	1475.09	3	4876.27	175	128.80	15	341.70		
2009	30	1449.93	3	4463.93	190	133.97	12	274.86		
2010	32	1613.94	3	4601.20	148	101.18	4	118.17		
2011	32	1518.18	2	3874.82						
2012	36	1695.11	3	3681.13						
2013	30	1336.66	3	2951.61						
2014	9	447.82								
2015										
2016										
2017										
2018										

Table A3. Data for MV-22 Osprey, SSN 774, Std Missile 2, T45TS, and Trident II
All Cost CY00$M

THIS PAGE INTENTIONALLY LEFT BLANK

APPENDIX B: FUNCTION PARAMETERS AND FIT

BASE MODEL MEASURE OF FIT AND PARAMETERS			
System	**Lot R^2_{adj}**	**T_1**	β
AAAV	0.57	7.81861	-0.06793
Abrams Upgrade	0.01	14.06907	-0.13987
Bradley Upgrade	0.88	15.74449	-0.28477
C-17	0.58	574.31966	-0.17606
CH-47F	0.95	20.84342	-0.20234
Crusader	0.91	16.86318	-0.17437
DDG 51	0.67	1121.24115	-0.04546
EELV	0.42	93.88424	-0.06959
FA-18 E/F	0.65	222.60646	-0.22781
JSOW	0.89	1.68594	-0.26177
Minuteman III	0.87	8.31990	-0.22178
NAVSTAR	0.55	119.00257	-0.15928
MV-22 Osprey	0.94	168.38496	-0.22447
SSN 774	0.75	2686.77836	-0.19130
Std Missile 2	0.79	34.58983	-0.54884
T-45TS	0.94	58.19118	-0.18214
Trident II	-0.35	77.10653	-0.15422
MEDIAN	**0.75**		
MEAN	**0.65**		

Table B1. Base Model R^2_{adj} and Fitted Parameters

MULTIPLICATIVE RATE MODEL MEASURE OF FIT AND PARAMETERS				
System	**Lot R^2_{adj}**	T_1	β	χ
AAAV	0.70	10.148	-0.04204	-0.08169
Abrams Upgrade	0.42	163.449	-0.07840	-0.62766
Bradley Upgrade	0.91	21.790	-0.13633	-0.24966
C-17	0.83	771.727	-0.06125	-0.35681
CH-47F	0.95	19.672	-0.18633	*0.00000*
Crusader	0.90	20.757	-0.15243	-0.06508
DDG 51	0.97	1510.120	-0.03590	-0.31790
EELV	0.88	95.516	-0.07395	*0.00000*
FA-18 E/F	0.90	541.808	-0.12197	-0.39134
JSOW	0.88	2.550	-0.23668	-0.09960
Minuteman III	0.94	17.501	-0.01105	-0.44964
NAVSTAR	0.53	114.538	-0.14171	*0.00000*
MV-22 Osprey	0.95	205.450	-0.15947	-0.16123
SSN 774	0.71	1967.715	-0.02312	-0.34216
Std Missile 2	0.79	39.154	-0.47226	-0.13194
T-45TS	0.93	60.082	-0.20338	0.01334
Trident II	0.75	151.321	-0.17933	-0.31996
MEDIAN	**0.88**			
MEAN	**0.81**			

Table B2. Multiplicative Rate Model R^2_{adj} and Fitted Parameters

IDA RATE-PENALTY MODEL MEASURE OF FIT AND PARAMETERS					
System	**Lot R^2_{adj}**	T_1	β	δ	**R***
AAAV	0.51	6.760	-0.04525	0.00553	200.000
Abrams Upgrade	0.03	17.338	-0.19414	0.04208	120.000
Bradley Upgrade	0.94	4.677	-0.11321	0.02098	190.548
C-17	0.87	322.087	-0.08388	19.90869	15.000
CH-47F	0.96	20.961	-0.20174	0.00100	26.000
Crusader	0.88	17.939	-0.18500	0.00947	240.000
DDG 51	0.94	1001.187	-0.06159	161.49438	4.923
EELV	0.89	95.430	-0.07374	0.00100	13.000
FA-18 E/F	0.56	261.427	-0.26459	0.00100	48.000
JSOW	0.82	0.857	-0.18994	0.00041	731.456
Minuteman III	0.85	2.251	-0.00036	0.05906	80.000
NAVSTAR	0.55	123.303	-0.16641	*0.00000*	3.000
MV-22 Osprey	0.95	159.058	-0.21551	0.46394	30.000
SSN 774	0.70	1484.113	*0.00000*	418.25901	3.000
Std Missile 2	0.80	17.657	-0.46028	0.00600	150.037
T-45TS	0.96	61.104	-0.19889	0.23579	12.000
Trident II	0.92	105.786	-0.46859	16.36435	8.933
MEDIAN	**0.87**				
MEAN	**0.77**				

Table B3. IDA Rate-Penalty Model R^2_{adj} and Fitted Parameters

LURP MODEL MEASURE OF FIT AND PARAMETERS					
System	**Lot R^2_{adj}**	T_1	β	δ	**R***
AAAV	0.60	6.753	-0.04671	0.00489	200.000
Abrams Upgrade	0.18	16.985	-0.19390	0.02440	120.000
Bradley Upgrade	0.88	15.221	-0.30396	0.00765	169.274
C-17	0.84	341.854	-0.12343	16.78857	13.754
CH-47F	0.96	20.331	-0.19360	0.01384	26.000
Crusader	0.97	10.853	-0.11166	0.01396	240.000
DDG 51	0.92	990.690	-0.08644	130.04896	4.678
EELV	0.89	95.426	-0.07373	0.00100	13.000
FA-18 E/F	0.56	261.403	-0.26458	0.00100	48.000
JSOW	0.82	1.427	-0.25164	0.00014	675.000
Minuteman III	0.75	10.488	-0.27319	0.00100	80.000
NAVSTAR	0.55	123.303	-0.16641	*0.00000*	3.000
MV-22 Osprey	0.97	154.603	-0.21421	0.47396	30.000
SSN 774	0.75	1451.865	*0.00000*	306.01032	3.000
Std Missile 2	0.74	28.683	-0.53104	0.00178	141.829
T-45TS	0.95	61.960	-0.20159	0.11271	12.000
Trident II	0.93	94.346	-0.64826	9.68116	9.243
MEDIAN	**0.84**				
MEAN	**0.78**				

Table B4. LURP Model R^2_{adj} and Fitted Parameters

RATE CHANGE MODEL MEASURE OF FIT AND PARAMETERS				
System	**Lot R^2_{adj}**	**T_1**	**β**	**δ**
AAAV	0.76	8.315	-0.08823	0.00330
Abrams Upgrade	-0.06	21.832	-0.21975	*0.00000*
Bradley Upgrade	0.79	20.851	-0.33651	0.00100
C-17	0.51	579.953	-0.17122	0.00100
CH-47F	0.96	20.717	-0.19727	0.00100
Crusader	0.86	21.992	-0.21566	0.00100
DDG 51	0.61	1259.813	-0.08714	*0.00000*
EELV	0.94	91.330	-0.08007	2.51267
FA-18 E/F	0.52	261.482	-0.26463	0.00100
JSOW	0.89	1.692	-0.25741	-0.00008
Minuteman III	0.33	9.261	-0.25777	0.03262
NAVSTAR	0.55	123.303	-0.16641	*0.00000*
MV-22 Osprey	0.93	176.229	-0.23353	0.05506
SSN 774	0.69	2529.798	-0.16309	91.60310
Std Missile 2	0.73	38.029	-0.56567	0.00100
T-45TS	0.97	60.445	-0.20098	0.31447
Trident II	0.58	107.595	-0.28407	0.00100
MEDIAN	**0.73**			
MEAN	**0.68**			

Table B5. Rate-Change Model R^2_{adj} and Fitted Parameters

BASE + OVERHEAD MODEL MEASURE OF FIT AND PARAMETERS				
System	**Lot R^2_{adj}**	T_1	β	Ω
AAAV	0.95	9.204	-0.10554	37.372
Abrams Upgrade	0.60	3.351	*0.00000*	219.906
Bradley Upgrade	0.93	7.003	-0.18030	53.401
C-17	0.84	315.273	-0.14253	831.878
CH-47F	0.96	21.320	-0.20451	1.000
Crusader	0.90	15.501	-0.17451	90.972
DDG 51	0.97	648.006	-0.01373	980.791
EELV	0.90	104.930	-0.09306	1.000
FA-18 E/F	0.89	111.095	-0.19579	1106.798
JSOW	0.90	1.498	-0.25403	4.665
Minuteman III	0.96	3.096	-0.08508	34.847
NAVSTAR	0.59	122.981	-0.16704	1.000
MV-22 Osprey	0.95	125.966	-0.19754	197.694
SSN 774	0.77	2419.285	-0.20507	414.191
Std Missile 2	0.81	27.305	-0.53731	16.248
T-45TS	0.95	66.235	-0.21480	1.000
Trident II	0.72	82.506	-0.29418	92.509
MEDIAN	**0.90**			
MEAN	**0.85**			

Table B6. Base + Overhead Model R^2_{adj} and Fitted Parameters

APPENDIX C: GAMS IMPLEMENTATION

```
$TITLE **PROCUREMENT SCHEDULE OPTIMIZATION MODEL (PSOM)**

*------DEFAULTS--------------------------------------------------
$OFFUPPER OFFSYMLIST OFFSYMXREF INLINECOM{ }

OPTIONS RESLIM = 100000
        ITERLIM = 100000
        LIMCOL = 0
        LIMROW = 0
        DECIMALS = 2
        SOLPRINT = OFF
        MIP = CPLEX
        SYSOUT = OFF
        OPTCR = .0005
;
*-------------------------------------------------------------
$ONTEXT
     Original:    9/28/00
     Author:      Donald E. Humpert

     Description: Base + Overhead Model
                  Time period one = current year = 2000;
                  Time period 19 = 2018;


$OFFTEXT
*---INDICES--------------------------------------------------

SET
  i           system
             /AAAV
              ABRAMS
              BRADLEY
              C17
              CH47F
              CRUSADER
              DDG51
              EELV
              FA18EF
              JSOW
              MMIII
              NAVSTAR
              OSPREY
              SSN774
              STDMSL2
              T45TS
              TRIDENT2/

  t           time periods / 1*19 /

  p           LRIP time periods / 1*3 /

  c           cost level /1*4/
;

ALIAS (t, tp);

*---DATA-----------------------------------------------------

PARAMETERS
  M(i)        LRIP time periods for system i
             /AAAV     1
              CRUSADER 1/

  QLRIP(i,p)  number of units i in LRIP period p
             /AAAV     .1     38
              CRUSADER .1     62/
```

```
CLRIP(i,p)  procurement cost of system i in LRIP period p
           /AAAV     .1     2.494
            CRUSADER .1     5.830/


INV(i)        starting inventory of system i in period 1
           /AAAV        0
            ABRAMS    848
            BRADLEY   229
            C17        85
            CH47F       0
            CRUSADER    0
            DDG51      48
            EELV        1
            FA18EF     98
            JSOW     1017
            MMIII     148
            NAVSTAR     0
            OSPREY     30
            SSN774      3
            STDMSL2   294
            T45TS     153
            TRIDENT2   35/


MINR(i)     min sustaining production rate for system i during FRP
           /AAAV       20
            ABRAMS     20
            BRADLEY    20
            C17         1
            CH47F       8
            CRUSADER   20
            DDG51       1
            EELV        4
            FA18EF     12
            JSOW      100
            MMIII      10
            NAVSTAR     3
            OSPREY      9
            SSN774      1
            STDMSL2    60
            T45TS       1
            TRIDENT2    5/


MAXR(i)     maximum production rate for system i during FRP
           /AAAV      200
            ABRAMS    120
            BRADLEY   235
            C17        15
            CH47F      29
            CRUSADER  240
            DDG51       3
            EELV       14
            FA18EF     48
            JSOW      900
            MMIII      80
            NAVSTAR     3
            OSPREY     36
            SSN774      3
            STDMSL2   190
            T45TS      15
            TRIDENT2   12/
```

```
EARLY(i)    earliest period in which FRP may begin for system i
           /AAAV        3
            ABRAMS      2
            BRADLEY     2
            C17         2
            CH47F       2
            CRUSADER    3
            DDG51       2
            EELV        2
            FA18EF      2
            JSOW        2
            MMIII       2
            NAVSTAR     2
            OSPREY      2
            SSN774      2
            STDMSL2     2
            T45TS       2
            TRIDENT2    2/


LATE(i)     latest period in which FRP may begin for system i
           /AAAV       16
            ABRAMS      2
            BRADLEY     2
            C17         2
            CH47F      14
            CRUSADER   18
            DDG51       2
            EELV        2
            FA18EF      2
            JSOW        2
            MMIII       2
            NAVSTAR    17
            OSPREY      2
            SSN774      2
            STDMSL2     2
            T45TS       2
            TRIDENT2    2/


D(i,t)      required quantities of system i by period t
           /AAAV     .10    950
            ABRAMS   .7    1131
            BRADLEY  .13   1602
            C17      .5     134
            CH47F    .14    300
            CRUSADER .18   1630
            DDG51    .6      58
            EELV     .19    181
            FA18EF   .11    548
            JSOW     .14  10000
            MMIII    .8     652
            NAVSTAR  .17     42
            OSPREY   .14    408
            SSN774   .15     30
            STDMSL2  .11   1500
            T45TS    .5     169
            TRIDENT2 .6      88/
```

```
  FIXED(i)     fixed costs for system i incurred if in frp
             /AAAV       37.372
              ABRAMS    219.906
              BRADLEY    53.401
              C17       831.878
              CH47F       1.000
              CRUSADER   90.972
              DDG51     980.791
              EELV        1.000
              FA18EF   1106.798
              JSOW        4.665
              MMIII      34.847
              NAVSTAR     1.000
              OSPREY    197.694
              SSN774    414.191
              STDMSL2    16.248
              T45TS       1.000
              TRIDENT2   92.509/;

TABLE  PRICE(i,c) price of system i at cost level c
                 1          2          3          4
  AAAV        6.2009     5.2218     5.8138     4.5611
  ABRAMS      3.3511     3.3511     3.3511     3.3511
  BRADLEY     2.4411     2.1905     2.0238     1.9015
  C17       165.7682   162.9792   160.3270   157.8955
  CH47F      12.7181     9.1060     7.7357     6.9360
  CRUSADER    7.5163     5.5858     4.8550     4.4248
  DDG51     614.1985   613.7836   613.3873   613.0102
  EELV       83.9663    73.7427    68.9474    65.8737
  FA18EF     42.3139    38.2048    35.3571    33.2098
  JSOW        0.4063     0.3388     0.2982     0.2701
  MMIII       1.9753     1.9019     1.8466     1.8027
  NAVSTAR   102.9337    82.8620    73.8571    68.0108
  OSPREY     56.4472    47.9924    43.0965    39.7409
  SSN774   1687.5765  1470.2191  1339.4992  1241.8150
  STDMSL2     1.1019      .8517     0.6912     0.5802
  T45TS      22.4033    22.2814    22.1632    22.0484
  TRIDENT2   27.7030    25.6831    24.0186    22.6609;


TABLE  LIMIT(i,c) min number of system i that must be procured at
 price c before any can be procured at price next c

           1          2          3          4
  AAAV     117        215        280        338
  ABRAMS   68         70         72         73
  BRADLEY  250        317        376        430
  C17      11         12         13         13
  CH47F    34         66         89         111
  CRUSADER 184        362        486        598
  DDG51    2          3          2          3
  EELV     24         41         52         63
  FA18EF   85         104        122        139
  JSOW     1510       2030       2500       2943
  MMIII    101        119        135        149
  NAVSTAR  6          9          12         15
  OSPREY   62         85         106        125
  SSN774   5          6          7          9
  STDMSL2  212        272        331        391
  T45TS    4          4          4          4
  TRIDENT2 11         13         14         15;

SCALAR   BGT annual budget /25000/;
```

```
*---VARIABLES---------------------------------------------
VARIABLE
  TOTAL       total procurement costs for all systems over all periods
;

POSITIVE VARIABLES
  q(i,t,c)    units of system i procured in period t at cost level c
  cumq(i,t)   cumulative units of system i procured by period t
  cost(i,t)   cost of units of system i procured in period t
;

BINARY VARIABLES
  start(i,t)  1 if system i begins FRP in period t else 0
  frp(i,t)    1 if system i is in FRP in period t else 0
  y(i,t,c)    1 if system i available at price level c in period t else 0
;

*---EQUATIONS-------------------------------------------------
   EQUATIONS
     OBJ            Objective Function
     PAYOUT(i,t)
     BUDGET(t)
     CUMQUANT(i,t)
     DEMAND(i,t)
     MINPROD(i,t)
     MAXPROD(i,t)
     EARLIEST(i)
     LATEST(i)
     NOBREAKS(i)
     FRPSTART(i,t)
     STEP1LHS(i,t,c)
     STEP1RHS(i,t,c)
     STEP2LHS(i,t,c)
     STEP2RHS(i,t,c)
     STEP3LHS(i,t,c)
     STEP3RHS(i,t,c)
     STEP4RHS(i,t,c)
   ;


*---OBJECTIVE FUNCTION---------------------------------------

   OBJ..                        TOTAL =e=  sum(i, sum(t, cost(i,t)));

*---CONSTRAINTS----------------------------------------------

   PAYOUT(i,t)..     cost(i,t) =e= sum(c, q(i,t,c)*PRICE(i,c)) + frp(i,t)*FIXED(i)
                               + sum(p$(ord(p)<=M(i)), CLRIP(i,p)*start(i,t+(1+M(i)-ord(p))));

   BUDGET(t)..       BGT  =g=  sum(i, cost(i,t));


   CUMQUANT(i,t)..   cumq(i,t) =e=  INV(i)
                                + sum(tp$(ord(tp)<=ord(t)),  sum(c, q(i,tp,c)))
                                + sum(tp$(ord(tp)<=ord(t)),  sum(p$(ord(p)<=M(i)),
                                  QLRIP(i,p)*start(i,tp+(1+M(i)-ord(p)))));

   DEMAND(i,t)..     cumq(i,t) =g= D(i,t);


   MINPROD(i,t)..    sum(c, q(i,t,c)) =g= frp(i,t)*MINR(i);


   MAXPROD(i,t)..    sum(c, q(i,t,c)) =l= frp(i,t)*MAXR(i);


   EARLIEST(i)..     sum(t, ord(t)*start(i,t)) =g= EARLY(i);
```

```
LATEST(i)..       sum(t, ord(t)*start(i,t)) =l= LATE(i);

NOBREAKS(i)..     sum(t, start(i,t)) =e= 1;

FRPSTART(i,t)..   start(i,t) =g= (frp(i,t) - frp(i,t-1));

STEP1LHS(i,t,c).. LIMIT(i,"1")*y(i,t,"1") =l= sum(tp$(ord(tp)<=ord(t)), q(i,tp,"1"));

STEP2LHS(i,t,c).. LIMIT(i,"2")*y(i,t,"2") =l= sum(tp$(ord(tp)<=ord(t)), q(i,tp,"2"));

STEP3LHS(i,t,c).. LIMIT(i,"3")*y(i,t,"3") =l= sum(tp$(ord(tp)<=ord(t)), q(i,tp,"3"));

STEP1RHS(i,t,c).. sum(tp$(ord(tp)<=ord(t)), q(i,tp,"1")) =l= LIMIT(i,"1");

STEP2RHS(i,t,c).. sum(tp$(ord(tp)<=ord(t)), q(i,tp,"2")) =l= LIMIT(i,"2")*y(i,t,"1");

STEP3RHS(i,t,c).. sum(tp$(ord(tp)<=ord(t)), q(i,tp,"3")) =l= LIMIT(i,"3")*y(i,t,"2");

STEP4RHS(i,t,c).. sum(tp$(ord(tp)<=ord(t)), q(i,tp,"4")) =l= LIMIT(i,"4")*y(i,t,"3");

*-----------------------------------------------------------

MODEL PSOM/ ALL /;
file report1;put report1;report1.pc=5;
put report1;
put'ANNUAL BUDGET','TOTAL COST'/;
put 'SYSTEM QUANTITIES PER PERIOD, PER BUDGET'//;
for(BGT = 22000 downto 12750 by 250,
   Solve PSOM using MIP MINIMIZING TOTAL;
   DISPLAY BUDGET.l;
   DISPLAY q.l;
   put report1;
   put BGT;
   put TOTAL.l;
   put/;
);
```

LIST OF REFERENCES

Air Force Institute of Technology, 1997, Course Reference for Intermediate Cost Analysis Course (BCE-204), School of Systems and Logistics, Dept of Quantitative Management, Wright-Patterson AFB, Ohio, Chapter 17.

Balut, S. J., Gulledge Jr., T. R., Womer, N. K., 1988, *An Improved Method for Repricing Systems*, Institute for Defense Analysis, IDA Paper P-2013, May.

Brooke, A., Kendrick, D., Meeraus, A and Raman R., 1997, GAMS: A User's Guide. The Scientific Press.

Gansler, J., 1999, Keynote Address to Advanced Program Management Course 99-02, Defense Systems Management College, Ft Belvior VA, 10 May.

Hamilton, 1992, *Regression With Graphics—A Second Course in Applied Statistics*, Duxbury Press, Belmont, CA.

Lee, D. A., 1997, *The Cost Analyst's Companion*, Logistics Management Institute, McLean, VA.

Loerch, A. G., 1999, "Incorporating Learning Curve Costs in Acquisition Strategy Optimization," Naval Research Logistics, v. 46, No. 3, pp. 255-271.

Rogers, P., 1999, Conversation between Phil Rogers, OUSD(AT&L), and the author, November 1999.

Weber, C. A., et al., 1999 "Portfolio Optimization Feasibility Study Mid-Project Review," Institute for Defense Analysis, July.

THIS PAGE INTENTIONALLY LEFT BLANK

INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center 2
   8725 John J. Kingman Road, Suite 0944
   Ft. Belvoir, VA 22060-6218

2. Dudley Knox Library 2
   Naval Postgraduate School
   411 Dyer Road
   Monterey, CA 93943-5101

3. Deputy Director, Resource Analysis 1
   OUSD(AT&L) / ARA
   Pentagon Room 3D161
   Washington DC 20301-3000

4. Director, Cost Analysis and Research Division 1
   Institute for Defense Analysis
   1801 N. Beauregard Street
   Alexandria, VA 22311-1772

5. Director, Training and Education 1
   MCCDC, Code C46
   1019 Elliot Road
   Quantico, VA 22134-5107

6. Director, Marine Corps Research Center 2
   MCCDC, Code C40RC
   1019 Elliot Road
   Quantico, VA 22134-5107

7. Director, Studies and Analysis Division 1
   MCCDC, Code C45
   300 Russell Road
   Quantico, VA 22134-5130

8. Director, Analysis Division 1
   MATCOM
   814 Radford Boulevard
   Albany, GA 31704-1128

9. Marine Corps Representative 1
   Naval Postgraduate School
   Code 037, Bldg. 330, Ingersoll Hall, Room 116
   555 Dyer Road
   Monterey, CA 93943

10. Marine Corps Tactical Systems Support Activity 1
Technical Advisory Branch
Attn: Librarian
Box 555171
Camp Pendleton, CA 92055-5080

11. Operations Analysis 1
Code 30
Naval Postgraduate School
Monterey, CA 93943-5000

12. Professor Alan R. Washburn 1
Code OR/WS
Naval Postgraduate School
Monterey, CA 93943-5000

13. LCDR Timothy P. Anderson 1
Code OR/AT
Naval Postgraduate School
Monterey, CA 93943-5000

14. Major Donald E. Humpert 3
167 Hickory Grove Road
Leesburg, VA 31763